PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

New Methods for the Analysis of Geomagnetic Fields and their Application to the Sq Field of 1932-3

A. T. Price and G. A. Wilkins

Phil. Trans. R. Soc. Lond. A 1963 **256**, 31-98
doi: 10.1098/rsta.1963.0016

NEW METHODS FOR THE ANALYSIS OF GEOMAGNETIC FIELDS AND THEIR APPLICATION TO THE Sq FIELD OF 1932–3

By A. T. PRICE
University of Exeter
and G. A. WILKINS
H.M. Nautical Almanac Office

(*Communicated by S. Chapman, F.R.S.—Received* 25 *October* 1962)

CONTENTS

New methods for interpolating and analyzing magnetic fields observed over the earth's surface are described and applied to the quiet-day magnetic daily variations during the sunspot minimum 1932–3. The methods avoid some of the disadvantages inherent in the use of spherical harmonic analysis. The field components are interpolated over the earth for selected instants of universal time. A potential is determined by numerical methods, the north and east components being used simultaneously. The potential is separated into parts of external and internal origin by the use of surface-integral formulae. The methods are described in part I, and tables required for their application are given. Part II deals with the results thus obtained for the Sq field for the combined months 1933 May to August. The definition of the Sq field and the reduction of the data to a standard form are also discussed.

I. METHODS FOR INTERPOLATION AND ANALYSIS OF SURFACE FIELDS

1. Survey of methods

1·1 *Introduction*

One method much used in the study of geomagnetic fields is spherical harmonic analysis, in which the potential of the particular magnetic field being investigated is represented by a series of spherical harmonic terms. This method has been of great value in the study of many geomagnetic phenomena; but there are some phenomena where alternative methods of analysis are desirable. This paper describes an attempt to develop some new methods of analysis, which are applied to the Sq field (i.e. the varying field corresponding to the daily magnetic variations on quiet (solar) days) deduced from observations during the International Polar Year 1932–3. Most of the work was done several years ago. It was described by one of us (Wilkins 1951), but a suitable computer was not at that time available for completing the heavy numerical work involved. This has now been done using the I.C.T. 1201 computer in H.M. Nautical Almanac Office. The new methods are described, with special reference to the Sq field, in part I. Their applications to the Sq field during 1932–3 are considered in part II. We believe that the methods may also prove useful in the study and analysis of other geophysical data, such as those obtained during the recent International Geophysical Year.

1·2. *The general nature of the Sq field*

On quiet days it is found that at stations in about the same latitude (except near the magnetic equator or in high latitudes) the magnetic elements deviate from their mean values by roughly the same amounts at corresponding local times. Thus the variations at the earth's surface can be regarded as being produced by a field that is approximately constant and stationary when viewed from the sun. This local-time part of the Sq field has been studied in great detail, especially by Chapman (1919; see also Chapman & Bartels 1940, chapter 20). The procedure has been to express the local-time variations at each station as a Fourier series (usually ending with the fourth harmonic). The Fourier coefficients thus found are averaged round circles of latitude. Then an expression for the potential is obtained as a series of spherical harmonics, which approximately corresponds to the distribution of these coefficients over the earth. That the field possesses a potential is inferred from the non-existence of any appreciable vertical electric current at the earth's surface. It has been confirmed (Chapman & Price 1928) by the calculation of line integrals of the field round a large closed circuit in Europe; there the field can be interpolated fairly accurately. Their values were found to be zero to within the probable errors of observation and interpolation. Important results relating to the main features of the Sq field have been obtained with the aid of spherical harmonic analysis, but the method is not well adapted for studying detailed features. McNish (1937) tried to take account of the enhanced Sq variations at Huancayo by making a spherical harmonic analysis of the data from five American stations, and he derived a corresponding ionospheric current distribution, but the results obtained are of doubtful significance.

The residual field obtained when the local-time field, determined in the above way, is subtracted from the observed field is found to be quite large. It has been studied by Benkova (1940) and by Maeda (1953) who have attempted, with only partial success, to represent it by a spherical harmonic expression. Such representations of the field as the *sum* of a strictly local-time part (corresponding to a constant field within which the earth is rotating) and a part depending on geographic position and universal time may be rather artificial. A more natural representation would seem to be one corresponding to a field stationary relative to the sun but varying somewhat in intensity and distribution with the angular position of the earth rotating within it.

1·3. *Spherical harmonic analysis*

In a spherical harmonic analysis the potential can be separately calculated for each component of the field. The east component is generally first used; then, as a check, the north component is derived from this potential and the calculated and observed values of this component are compared. For the Sq field quite considerable discrepancies are usually found between them. These discrepancies may be due in part to various sources of error in the reduced observations, i.e. in the seasonal mean values of Sq; these sources of error are discussed later in §4·33. But probably the discrepancies are mainly due to the inadequacy of the limited number of spherical harmonics chosen to represent the field, and to incorrect assumptions about its variation with time. Hasegawa (1936) and Hasegawa & Ota (1950) attempted to avoid some of these difficulties by using graphical and numerical methods to interpolate the field over the earth at a number of separate epochs, before making a spherical harmonic analysis of the field for each epoch. Their methods are discussed in detail in §2·21.

One disadvantage of spherical harmonic analysis is that only a reasonably smooth world-wide field can be represented satisfactorily by the necessarily limited number of harmonics that can be used. Local features either get smoothed out and obscured, or, if they are represented by additional harmonic terms, may introduce errors into the representation of the field elsewhere. For example, the great enhancement of the Sq field found near the magnetic equator cannot be represented conveniently by a spherical harmonic expression.

Although the north and east components have so far always been analyzed separately this is not an essential disadvantage of the method of spherical harmonic analysis. The coefficients for the potential could be determined from a least-squares solution in which the equations of conditions for both north and east components were used in forming the normal equations. Further, the potential at selected epochs of universal time could be determined by using the instantaneous values and (time) derivatives of both field components; it would certainly be adequate to assume that the instantaneous value of the derivative with respect to longitude was equal to the observed derivative with respect to time. The number of equations of conditions would be four times the number of observatories; it should therefore be possible to determine at least the principal terms of the expansions, provided that disturbing effects were not too great and that many observatories were working at the chosen epochs. The main difficulties would arise in the separation of the potential into parts of external and internal origin, since this depends to a large extent on the knowledge of the variations of the vertical component; it seems that these are less well determined and are also more disturbed by geological irregularities which affect the induced earth currents that are responsible for the internal part of the Sq field. It would also be of interest to attempt to analyze all three components together and so obtain the coefficients of the external and internal parts of the potential in one solution.

1·4. *The new methods*

Spherical harmonics express an important part of the field in analytic form suitable for theoretical work, such as the separation of the field into parts of external and internal origin. In the methods we have developed, the advantage of an analytic expression is sacrificed, and the separation of the field is much more laborious, since it involves the numerical evaluation of a large number of surface integrals. This, however, is less important than formerly, now that the burden of heavy numerical work can be greatly eased by modern computers. Moreover, besides avoiding some of the difficulties already mentioned, the new methods lead to a potential which corresponds much more closely to the observed field. They have also other practical advantages that are worth noting.

First, they make it possible to use the north and east components simultaneously to obtain the best approximation to the potential, and also to take account of the varying reliability of the data. Secondly, whereas in spherical harmonic analysis the interpolation is automatic and simultaneously world-wide, in the new methods it is essentially local in character and local features can be studied in detail and can be included without the risk of introducing incorrect modifications of the field in other areas. Lastly, the results can be modified in the light of further information about the field which may become available later without requiring complete recalculation of the potential of the entire field, as is usually necessary in spherical harmonic analysis.

The new methods give for selected epochs of universal time the value of the field components and of the external and internal contributions to the potential at each of a network of points distributed evenly over the earth's surface. Such sets of values are more readily fitted by a spherical harmonic expression than are the observed variations at unevenly distributed stations. Although this fitting has still to be attempted it is expected that such expressions would give a much better representation of the observed field.

The work proceeds in three principal stages. In the first stage the field components are interpolated graphically at selected instants of universal time. This interpolation involves determining certain curves which we term 'lines of corresponding latitude for Sq'. In the next stage, the total potential over the earth at the chosen instants is determined by a numerical method which uses simultaneously the north and east components. The original estimates of these components are then modified until they are compatible with the existence of a potential while still giving a reasonable fit to the original observations. The fact that this can be done justifies the original assumption of the existence of a potential. Finally, the potential is separated into parts corresponding to fields of external and internal origin, by using a surface-integral formula first discussed by Vestine (1941). The integrals are evaluated with the aid of the tables given here (table 1, p. 47). The calculations are easily programmed for a computer.

The method of separating the field into parts of external and internal origin can be easily modified to investigate fields given over a plane surface, e.g. local geomagnetic fields over a part of the earth's surface sufficiently small to be treated as a plane. A table of the values necessary for evaluating the corresponding surface integrals in this case is also given (table 2, p. 51).

2. The determination of the field

2·1. *The interpolation of the field components*

The daily variations of the magnetic elements are observed at a set of stations irregularly distributed over the earth. From them we wish to obtain as accurate a picture as possible of the seasonal mean Sq field over the whole earth at selected instants of universal time (U.T.). To achieve this it is desirable to have the data for each observatory in a standard form, that is as deviations from the night values at each exact hour of local mean time (L.T.) The mean variations should be derived from the same selected quiet days and should be corrected for non-cyclic change in the same way.

The interpolation is fairly simple because the Sq field is largely a local-time field, i.e. it is approximately constant when viewed from the sun. This is, however, only a first approximation, and some features of the daily variations may change appreciably as we move round a circle of geographic latitude. By comparing the variations at neighbouring observatories one can construct curves along which the changes of the variations are least; these we term 'lines of corresponding latitude for Sq'. They are found to follow approximately, but not exactly, the lines of equal magnetic dip; a map showing these lines for the J-months (May to August) of 1933 has already been given by us (1951) and is given here in a slightly modified form as figure 7 in part II. In constructing such maps it is possible to take into account information about the Sq field obtained in other years. A preliminary interpolation of the field over the earth is now made by assuming that the local-time variations at nearby points along a line of corresponding latitude are approximately equal. This approximation appears to be generally satisfactory for points differing in longitude by up to 30°, or in some cases 60° or more, depending on the geographic position and the component to be interpolated. Hasegawa (1936) apparently assumed that the local-time variations are approximately constant along geographic-latitude circles, but this is not satisfactory, especially in equatorial regions.

It is convenient to obtain for each selected instant the values of the field components at a set of evenly distributed points referred to as mesh-points. These are chosen to lie at the intersections of a set of hour-meridians at intervals of 15° of longitude (i.e. at the exact hours of longitude east of Greenwich), and a set of mesh-circles at intervals of 10° of latitude. By interpolating along the lines of corresponding latitude defined above, the observed variations at any station are used to estimate the field at any particular instant at one point on each of the nearby hour-meridians. This is referred to as the primary interpolation. The field is then interpolated graphically along each hour-meridian, and afterwards along each mesh-circle; this is referred to as the secondary interpolation.

Let O be an observatory at geographic longitude λ^{h}E and P a point at longitude $\lambda^{*\mathrm{h}}$E on the line of corresponding latitude through O. Then the lines of corresponding latitude are such that the estimate of the field at P at t^{h} U.T. is equal to the field at O at t'^{h} L.T., where t' equals the local mean time at P at t^{h} U.T., i.e. $t' = t+\lambda^*$. It is convenient to arrange that λ^* and t shall be integers and in this case t' is also an integer; this is done by obtaining the daily variations at each observatory at the exact hours of local mean time, rather than at the exact hours of U.T. The primary interpolation then gives, for any particular instant of U.T., a set of primary estimates of the field components at certain points on each hour-meridian, the latitude of the points depending on the positions of the nearby observatories and on the lines of the corresponding latitude. The errors in these estimates depend on the rate of change of the variations along the line of corresponding latitude and on the difference of longitude between the observatory and the point. They will also, of course, depend on the accuracy of the original observations.

A smooth, reasonably fitting curve is drawn through these primary estimates, of which there are usually ten to fifteen on each hour-meridian. This gives an estimate of the field at each of the mesh-points. These secondary estimates are then plotted as a function of longitude along the mesh-circles. The curves so obtained are roughly similar to the local-time variations at the observatories. The secondary estimates are then modified until both sets of curves are smooth, show a gradual change of amplitude, phase and type with longitude and latitude, and fit the primary estimates reasonably well. It is then possible to draw the isomagnetic lines of each component. These may suggest further slight modifications of the secondary estimates.

The assumption that the field at the earth's surface is derivable from a potential leads to a compatibility condition that must be satisfied by the north and east components of the field. This gives a further check on the interpolation of the horizontal components, and may require further modification of the earlier estimates of the values at the mesh-points. No similar check can be applied to the vertical component. An important feature of our method is that the uncertainties in the data are immediately localized and made apparent, whereas they tend to be obscured in 'fixed-field' methods like that of Hasegawa & Ota, and in the method of spherical harmonic analysis.

2·2. *The determination of the field potential*

2·21. *Fixed-field methods*

When the north and east components of the field are independently interpolated over the earth, the estimated values are not immediately compatible with the existence of a potential. This is due partly to the errors in the observations, but probably much more to the uncertainties of the interpolation. In the method of spherical harmonic analysis the observations of Sq are used *directly* and the interpolation over the earth is automatic, but the two horizontal components are usually analyzed independently, and the results obtained then show considerable discrepancies. In an attempt to overcome this Hasegawa

& Ota (1950) first made graphical interpolations of the north and east components independently, and then derived a potential which best fitted both components according to a particular way of assessing the error.

This method is one of the 'fixed-field' methods in which the east component (Y) is assumed known along each of a set of circles at colatitudes θ_i ($i = 0, 1, 2, \ldots, I-1$) and the north component (X) is assumed known between colatitudes θ_0 and θ_{I-1} along a set of meridians at longitudes λ_j ($j = 0, 1, 2, \ldots, J$), the meridians λ_0 and λ_J being identical.

Let X_j (θ) denote the true value of X at (θ, λ_j) and $\hat{X}_j(\theta)$ the value estimated from the preliminary interpolation, and similarly let $Y_i(\lambda)$ and $\hat{Y}_i(\lambda)$ denote the values of Y. Let Ω_{ij} denote the true value of the potential Ω at (θ_i, λ_j), $\Omega_{ij}(\hat{X})$ its value determined from $\hat{X}$, $\Omega_{ij}(\hat{Y})$ its value determined from $\hat{Y}$, and $\hat{\Omega}_{ij}$ the final estimate of Ω. Then, denoting the radius of the earth by a,

$$\Omega_{ij} = \Omega_{0j} + \int_{\theta_0}^{\theta_i} X_j a \, d\theta = \Omega_{i0} - \int_{\lambda_0}^{\lambda_j} Y_i a \sin \theta_i \, d\lambda, \tag{2·1}$$

$$\Omega_{ij}(\hat{X}) = \Omega_{0j} + \int_{\theta_0}^{\theta_i} \hat{X}_j a \, d\theta, \tag{2·2}$$

and similarly for $\Omega_{ij}(\hat{Y})$.

Owing to errors of observation and interpolation, $\Omega_{ij}(\hat{X})$ and $\Omega_{ij}(\hat{Y})$ differ, and the estimate of Ω_{ij} is taken as

$$\hat{\Omega}_{ij} = \frac{w_x \Omega_{ij}(\hat{X}) + w_y \Omega_{ij}(\hat{Y})}{w_x + w_y} \quad (i > 0, j > 0), \tag{2·3}$$

where w_x and w_y are weighting factors depending on the reliability of $\hat{X}$ and Y.

Denoting the errors in $\Omega_{ij}(\hat{X})$ and $\Omega_{ij}(\hat{Y})$ by α_{ij} and β_{ij}, we have

$$\Omega_{ij} = \Omega_{ij}(\hat{X}) - \alpha_{ij} = \Omega_{ij}(\hat{Y}) - \beta_{ij}. \tag{2·4}$$

We also write

$$V_{ij}(X) = \int_{\theta_0}^{\theta_i} \hat{X}_j a \, d\theta, \quad V_{ij}(Y) = -\int_{\lambda_0}^{\lambda_j} \hat{Y}_i a \sin \theta_i \, d\lambda, \tag{2·5}$$

$$x_j = \hat{\Omega}_{0j}, \quad y_i = \hat{\Omega}_{i0}, \tag{2·6}$$

and we can take

$$x_0 = y_0 = \Omega_{00} = 0, \tag{2·7}$$

since the potential is arbitrary to the extent of a constant.

We then have from the above questions

$$\Omega_{ij} = x_j + V_{ij}(X) - \alpha_{ij} = y_i + V_{ij}(Y) - \beta_{ij}, \tag{2·8}$$

in which the quantities $V_{ij}(X)$, $V_{ij}(Y)$ are known and x_j, y_i have to be chosen in a suitable way.

The method used by Hasegawa & Ota was first to obtain from the above equations the expressions for x_j, y_i:

$$x_j = \{V_{ij}(Y) - V_{i0}(Y)\} - \{V_{ij}(X) - V_{i0}(X)\} - \{\beta_{ij} - \beta_{i0} - (\alpha_{ij} - \alpha_{i0})\}, \tag{2·9}$$

$$y_i = \{V_{ij}(X) - V_{0j}(X)\} - \{V_{ij}(Y) - V_{0j}(Y)\} - \{\alpha_{ij} - \alpha_{0j} - (\beta_{ij} - \beta_{0j})\}. \tag{2·10}$$

Ignoring for the moment the error terms (α, β), these give a set of I values for *each* of the x_j and a set of J values for *each* of the y_i. Owing to the errors the values in each set differ, and the mean value of each set is chosen as the final estimate of the true value. These mean values may then be used to calculate $\Omega_{ij}(\hat{X})$ and $\Omega_{ij}(\hat{Y})$ from equation (2·2) and hence $\hat{\Omega}_{ij}$ from equation (2·3).

The scatter in each set of values of x_j and y_i, and the differences between $\Omega_{ij}(\hat{X})$ and $\Omega_{ij}(\hat{Y})$, give an indication of the reliability of the final estimate of the field potential. Hasegawa & Ota (1950) state that there were appreciable differences in their calculated values of $\Omega_{ij}(\hat{X})$ and $\Omega_{ij}(\hat{Y})$, but give no details; they used the arithmetic mean of the two values (at each mesh-point) for their final estimate of Ω_{ij}.

An alternative method of dealing with a fixed field is to choose the values of x_j and y_i so that the mean square of the differences

$$\delta_{ij} = \Omega_{ij}(\hat{X}) - \Omega_{ij}(\hat{Y}) = \alpha_{ij} - \beta_{ij} \tag{2·11}$$

is as small as possible, i.e. so that

$$\frac{1}{IJ}\sum_{i,j}\delta_{ij}^2 \tag{2·12}$$

is a minimum. This leads to the two independent sets of simultaneous equations

$$x_j - \frac{1}{J}\sum_{j'} x_{j'} = \frac{1}{IJ}\sum_{i'}\sum_{j'} D_{i'j'} - \frac{1}{I}\sum_{i'} D_{i'j}, \tag{2·13}$$

$$y_i - \frac{1}{I}\sum_{i'} y_{i'} = \frac{1}{IJ}\sum_{i'}\sum_{j'} D_{i'j'} - \frac{1}{J}\sum_{j'} D_{ij'}, \tag{2·14}$$

where

$$D_{ij} = V_{ij}(X) - V_{ij}(Y). \tag{2·15}$$

These simultaneous equations can be solved for x_j and y_i with the aid of a computer. The resulting values are then substituted in equation (2·2) to obtain corresponding estimates $\Omega_{ij}(\hat{X})$ and $\Omega_{ij}(\hat{Y})$ for the potential. This method would give the narrowest limits to the potential corresponding to a *given* interpolated field, i.e. to a 'fixed field', but it is doubtful whether the extra labour involved would be justified. It should be noted that δ_{ij} is not equal to the error of the final estimate of the potential since, e.g. this would be $\frac{1}{2}(\alpha_{ij}+\beta_{ij})$ when $w_x = w_y$. This simply means that errors could still be present even if the estimates $\Omega_{ij}(\hat{X})$ and $\Omega_{ij}(\hat{Y})$ agreed exactly; in this case, however, one would not expect such errors to be serious.

2·22. *The method of residuals*

One disadvantage of the fixed-field methods described above is that any departure of the originally estimated field from a potential field is discovered only after completing a lengthy calculation for the entire field. Even then it is not easy to see how and where the estimated values of the field could be improved. It is clearly desirable to have a method whereby the estimated values of the field in any region can be adjusted so as to satisfy the condition for the existence of a potential, while remaining within tolerable limits of the field deduced from observations. Our 'method of residuals' has been developed with this aim in view. It is based on the condition that the line integral of magnetic force around any closed circuit in a potential field is zero. The 'residuals' are the values of the line integrals of the estimated field around contours defined by the lines of an orthogonal mesh, and the method proceeds by systematically adjusting the estimates of the field so as to reduce all the residuals to zero.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The method has several other advantages. The mesh can readily be made finer or coarser in any region according to the availability and reliability of the data; this can be done at any stage in the analysis, and for any limited part of the surface, without affecting the results in other parts. The regions where the two components are compatible are shown immediately. The accuracy of the potential function in these regions is not affected by divergencies in other regions, as is generally the case with fixed-field methods and with spherical harmonic analyses. The calculated potential in any region can be readily corrected to take account of observations which may later become available, without requiring complete recalculation of the potential for the entire field, as in other methods.

Using the same mesh as in §2·21 we consider the line integral of the field around a typical mesh-circuit ABCD shown in figure 1. We write

$$\delta_1 = \Omega_B - \Omega_A = -\int_{\lambda_j}^{\lambda_{j+1}} Y_i a \sin\theta_i \, d\lambda, \tag{2·16}$$

$$\delta_2 = \Omega_C - \Omega_B = \int_{\theta_i}^{\theta_{i+1}} X_{j+1} a \, d\theta, \tag{2·17}$$

$$\delta_3 = \Omega_C - \Omega_D = -\int_{\lambda_j}^{\lambda_{j+1}} Y_{i+1} a \sin\theta_{i+1} \, d\lambda, \tag{2·18}$$

$$\delta_4 = \Omega_D - \Omega_A = \int_{\theta_i}^{\theta_{i+1}} X_j a \, d\theta. \tag{2·19}$$

The values of the potential differences δ_1, δ_2, ... are calculated from the estimated mean values over AB, BC, ... of Y and X for all the mesh-circuits. These mean values are obtained from the graphical interpolation of the field components described in §2·1, and are multiplied by corresponding factors $-ah_\lambda \sin\theta_i$, ah_θ, etc., to give the corresponding potential differences, where h_λ and h_θ are the intervals of longitude and colatitude.

A convenient unit of potential is 10^3 gilberts (1 gilbert = 1G-cm) since, as we now show, the difference of potential between adjacent mesh-points is then of the same order of magnitude as the mean value of the component of force along the line joining them. The unit of field strength, or rather of induction of air, is the gamma (γ), rather than the gauss (G), where $1\gamma = 10^{-5}$ G. The mean radius (a) of the earth is $6·37 \times 10^8$ cm and a suitable mesh has intervals of longitude (h_λ) equal to 15° or $\pi/12$ radians and intervals of colatitude equal to 10° or $\pi/18$ radians.

Hence, if $\bar{X}$ and $\bar{Y}$ are the mean values of the north and east components, expressed in gammas, the corresponding potential differences (δ_X, δ_Y) between mesh-points are given by

$$\delta_X = ah_\theta \bar{X} = 1·11\,\bar{X} \times 10^3 \text{ gilberts},$$
$$\delta_Y = a \sin\theta h_\lambda \bar{Y} = 1·67 \sin\theta \bar{Y} \times 10^3 \text{ gilberts}.$$

For each circuit the value of the 'residual' R, defined by

$$R = (\delta_1 + \delta_2) - (\delta_3 + \delta_4), \tag{2·20}$$

is calculated. Its value is zero if the estimates of X and Y are compatible with the existence of a potential. The first estimates of X and Y do not in general lead to $R = 0$ everywhere. They have therefore to be modified. Each δ is used in calculating the residuals in two adjacent circuits, and any change in its value alters these two residuals by equal and opposite amounts. The sum of the residuals over the entire earth must be zero since the surface is closed. When the condition $R = 0$ is satisfied for all the circuits, the potential at all the mesh-points can be calculated, its value at any one chosen point being fixed arbitrarily. It is convenient to do the work in two stages.

In the first stage a coarse mesh is used, the latitude circles being drawn at intervals of 20° and the meridians at 30° of longitude. Each mesh-circuit, e.g. ABCD in figure 1, is divided into four compartments. The first estimates of δ_3 and δ_4 are inserted in the compartments indicated in figure 1, δ_1 and δ_2 being placed in the corresponding compartments of the adjacent circuits as indicated. The residuals are then calculated and placed in the top right-hand compartment, the lower left-hand compartment being reserved for the potential itself. The reduction of the residuals then begins. This process is not entirely

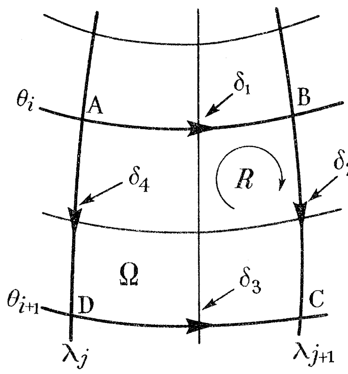


FIGURE 1. Arrangement of quantities in the method of residuals.

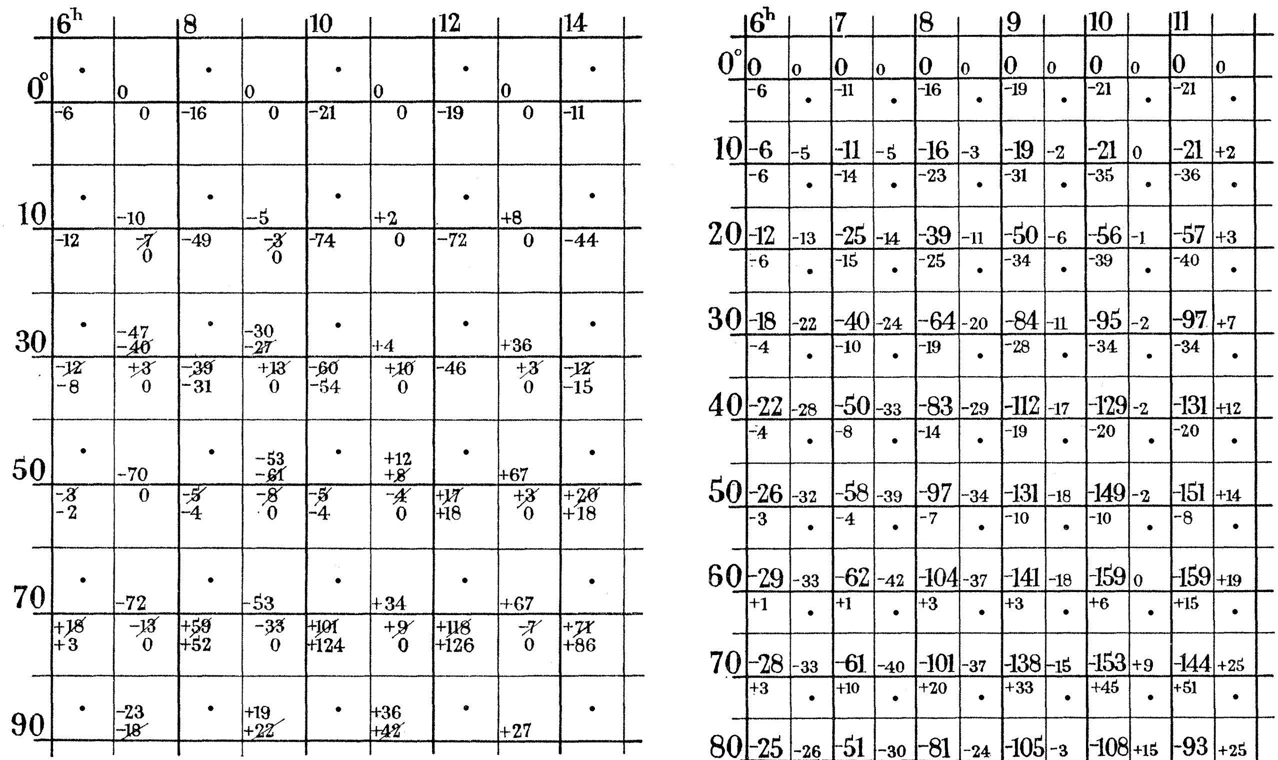
automatic, and the resulting potential is not uniquely determinate. Personal judgements are involved, and these must influence the final results. The aim is to obtain the simplest potential consistent with the observed field. The process is easiest when the residuals in a region are of mixed sign. Sometimes it is difficult to see what to do in regions where the residuals are predominantly of one sign, without departing too far from the field deduced from the observations. Such cases occur where the distribution of observatories is sparse, and the interpolation mesh is not fine enough to take into account some distinctly local feature of the field. Alternatively the reduction of the observations may not have completely eliminated some extraneous transient field.

An example of the first stage of the reduction of the residuals is shown in figure 2 (*a*); only the first and final estimates of the potential differences are shown. This example is taken from the determination of the potential at 4^{h} U.T., J-months, 1933.

In the second stage the mesh is made finer, intervals of 10° of latitude and 15° longitude being used. The potential differences found at the end of the first stage are divided into two parts, and the values on the new mesh-lines are chosen in such a way that the residuals are zero. These values are then checked with the original observations and altered, where possible, to give a better agreement. The values of the total potential at the new mesh-points are then found and inserted as in figure 2 (*b*). It is necessary to ensure that the modifications made at this stage do not introduce any non-zero residuals, i.e. the pattern of field changes must be compatible with the existence of a potential. Hence modifications are limited to those which correspond to changes in the potential at a few points. The most useful pattern is that in which the potential at only one point is changed, but composite patterns may also be built up.

There are no complications at the poles; a rectangular representation of the mesh is still permissible, provided that the potential differences between the ends of the meridians at the poles are all taken as zero.

In applying the method it is sometimes necessary to judge the relative reliability of the original estimates of the north and east components, when it is found that they cannot both be fitted to a potential. Thus the component having the least scatter in the primary-interpolation points would be given most weight, or used alone except when this means



(*a*) Initial reduction with coarse mesh. (*b*) Final results with fine mesh.

FIGURE 2. Examples of the two stages in the method of residuals.

departing completely from the values of the other component. It should be observed that the potential differences deduced from the east component on the latitude circles cannot be compared directly with the values of this component at the primary-interpolation points which are on the meridians. Fitting a potential to keep the east component a smooth function of longitude may result in values of this component which depart a little from those at the primary-interpolation points. But such departures as are found are much less serious than the discrepancies found in the other methods of analyzing the Sq field. We have found it possible by the method of residuals to fit a potential to the observed horizontal components in middle and low latitudes with a degree of accuracy which is reasonable in view of the likely uncertainties in the reduced observations and in the interpolation.

There does not seem to be any advantage in attempting to use an automatic computer for this part of the work. The numerical work is small once the field data have been expressed in a suitable form and tables giving the potential differences as functions of the force components have been prepared. Indeed the use of a computer is likely to be disadvantageous, since it would necessitate the formulation of rules that would define the smoothness of the interpolation and would determine the manner in which the adopted field could depart from the primary estimates. Errors of judgement by the investigator in formulating these rules would have a more profound effect on the resulting potential than would an occasional mistaken decision as to the value to adopt when the primary estimates are incompatible. In fact, the use of such rules might hide new and interesting features of the field that might otherwise be noticed by the investigator when attempting to find the potential.

3. The separation of the field

3·1. *Surface-integral methods*

When a magnetic field, given over a closed surface, is derivable from a scalar potential, as in the case of the Sq field, it must originate from sources outside or inside the surface, and no electric currents can cross the surface. A knowledge of the vertical component of the field over the surface together with the potential derived from the horizontal components is sufficient to separate the field into the parts of external and internal origin. In previous investigations this separation has always been made by a spherical harmonic analysis, the disadvantages of which have already been mentioned.

Vestine (1941) suggested a method and derived some formulae for separating the field by using certain surface integrals. His work was later extended by Taylor (1944) and Davids (1944). As far as we know the evaluation of the integrals involved has not been considered except for one or two special cases of the eccentric dipole field of the earth. We have considered the evaluation of these integrals in some detail, as we think the method may prove to be of considerable value. Vestine deduced a number of integral formulae for a spherical surface by using certain expansions in spherical harmonics. Some of these expansions are not convergent, and some of the formulae need correction. However, it is sufficient for our present purpose to begin with the result (see, for example, Vestine 1941, equation 19)

$$(\Omega_e-\Omega_i)_A=\Omega_0+\frac{1}{2\pi}\int_S\left\{\frac{1}{R}\frac{\partial}{\partial n}(\Omega-\Omega_0)-(\Omega-\Omega_0)\frac{\partial}{\partial n}\left(\frac{1}{R}\right)\right\}\mathrm{d}S, \qquad (3·1)$$

where Ω_e, Ω_i are the parts of external and internal origin of the potential Ω; Ω_0 is a constant; $\partial/\partial n$ denotes differentiation along the outward normal to the surface S, and R is the distance from the point A on the surface to the element of surface $\mathrm{d}S$.

In practice the formula is simplified by taking $\Omega_0=0$, but it may be noted that when the potential function Ω over the surface is determined from the tangential components of force it is arbitrary to the extent of a constant. Let therefore Ω and Ω' be two potentials which differ by the constant ω, so that

$$\Omega=\Omega'+\omega \quad \text{and} \quad \Omega_e+\Omega_i=\Omega'_e+\Omega'_i+\omega. \qquad (3·2)$$

Then from equation (3·1) it follows that

$$\Omega_e-\Omega_i=\Omega'_e-\Omega'_i+\omega, \quad \text{since} \quad \int_S\frac{\partial}{\partial n}\left(\frac{1}{R}\right)\mathrm{d}S=-2\pi; \qquad (3·3)$$

and hence $$\Omega_e = \Omega'_e + \omega \quad \text{and} \quad \Omega_i = \Omega'_i. \tag{3·4}$$

Thus the arbitrary constant is contained in Ω_e, and Ω_i is completely determinate, in agreement with the assumption implicit in the derivation of equation (3·1) that Ω_i vanishes at infinity.

3·2. *The application to a sphere*

When the surface S is a sphere the equation (3·1) can be reduced to the form

$$(\Omega_e - \Omega_i)_A = \int_S (\Omega + 2aZ) \frac{dS}{4\pi aR}, \tag{3·5}$$

where Z is the inward normal component of force. Since the field is not now given as an analytic function, this integral must be evaluated by numerical or graphical methods. The method suggested by Vestine and used by Taylor and Davids is as follows.

The point A is taken as the pole of a system of spherical co-ordinates so that, if $\overline{\Omega} + 2a\overline{Z}$ denotes the mean value of $\Omega + 2aZ$ round any circle of colatitude θ,

$$(\Omega_e - \Omega_i)_A = \int_0^{\frac{1}{2}\pi} (\overline{\Omega} + 2a\overline{Z}) \cos\psi \, d\psi; \; \psi = \tfrac{1}{2}\theta. \tag{3·6}$$

If the values of $\overline{\Omega} + 2a\overline{Z}$ are found for a sufficient number of small circles, this integral can be readily calculated. The evaluation of $\overline{\Omega} + 2a\overline{Z}$ is not, however, very convenient, since the co-ordinates with reference to which the field is given do not in general coincide with those required in this evaluation. Thus in a geomagnetic application the field is usually given on charts drawn to geographic (or sometimes geomagnetic) co-ordinates. To apply the above method it would be necessary to refer the field by interpolation to a new set of co-ordinates for each separation point A. The transformation to the new co-ordinates could be made by using special transformation tables, as suggested by Davids, or by using a set of transformation charts drawn on tracing paper. Both ways would, however, prove very laborious when applied to many separation points. The following alternative method of using equation (3·5) has therefore been adopted.

The spherical surface is divided into a large number of tesserae by a co-ordinate mesh corresponding to that used in defining the field. Let ΔS_N denote a typical tesseral element centred at the point Q_N, and I_N^A the 'basic integral' defined by

$$I_N^A = \int_{\Delta S_N} \frac{dS}{4\pi a R_A}, \tag{3·7}$$

where R_A is the distance of dS from the point A. Then equation (3·5) is replaced by the approximation

$$(\Omega_e - \Omega_i)_A \simeq \sum_N (\Omega + 2aZ)_N I_N^A, \tag{3·8}$$

where the summation of the products is taken over the whole sphere, and where $(\Omega + 2aZ)_N$ is the value of $\Omega + 2aZ$ at the mesh-point Q_N, or of its mean value over the element if the field varies appreciably over the element. One table of basic integrals (I_N^A) is sufficient for each possible value of the latitude of A since the values of I_N^A depend on the differences between the longitudes of A and Q_N. The evaluation of expression (3·8) essentially involves the scalar product of two vectors whose components are the values of the field at the mesh-points and the values of the corresponding basic integrals. This scalar product can itself be formed as the sum of the scalar products of appropriate pairs of subvectors, whose components are the values for one meridian. The evaluation can be conveniently carried out by using an automatic computer, although the precise procedure depends largely on the facilities available.

The numerical value of the factor $2a$ in the expression $\Omega+2aZ$ is equal to 12·74 when we adopt 1γ and 10^3 gilberts for the units of Z and Ω respectively (for the reasons given in 2·22). Since errors of 1 unit in X or Y correspond to errors of about 1 unit in Ω it is clear that the quantity $\Omega+2aZ$ is much more sensitive to errors in Z than it is to errors in X and Y.

The method depends on the evaluation of the tables of basic integrals, but once the tables have been calculated they can be applied to any other fields. The value of the integral (3·7) depends on the positions of the points A and Q_N and on the size of the tesseral elements over which the integral is to be evaluated. Let (θ_A, λ_A) denote the co-ordinates of A, (θ_N, λ_N) those of Q_N and (θ, λ) those of Q, the integration point. Suppose the tessera to be defined by the latitude circles at colatitudes $\theta_N-\alpha$ and $\theta_N+\alpha$, and by the meridians at longitudes $\lambda_N-\beta$ and $\lambda_N+\beta$. The values of α and β are conveniently taken to be the same for all tesserae, even though the areas of the tesserae are much smaller at the poles than at the equator. The integral over the element surrounding Q_N is given by

$$I_N^A = \int_{\Delta S_N} \frac{dS}{4\pi a R_A} = \int_{\lambda_N-\beta}^{\lambda_N+\beta}\int_{\theta_N-\alpha}^{\theta_N+\alpha} \frac{\sin\theta\, d\theta\, d\lambda}{8\pi \sin\gamma}, \tag{3·9}$$

where 2γ is the angle AOQ, O being the centre of the sphere; its value is independent of the radius of the sphere. It is only necessary to evaluate the integrals for $\lambda_A = 0$ and for $0 \leqslant \theta_A \leqslant 90°$, since all other values can be obtained from considerations of symmetry. Further, it is easily shown that the value of the integral taken over the whole sphere is unity; this provides a useful summation check on the values for each point A. The calculation of the basic integrals is considered further in § 3·3.

There are several sources of error in the determination of Ω_e and Ω_i by the above method. These include: (i) observational and interpolation errors in the values of Ω and $\Omega+2aZ$; (ii) errors due to the use of a finite sum to replace the surface integral (3·5); (iii) errors in the values of the basic integrals; and (iv) cumulative errors in the evaluation of the scalar products. All these must be taken into account in estimating the probable error of the final result: we consider them in turn.

(i) The relative contributions of Ω and $2aZ$ to their sum vary with the nature of the field and with position. It has been shown above that the errors in the vertical component (Z) have a greater effect than similar errors in the horizontal components.

(ii) The errors due to the use of a finite sum to approximate to the integral over the sphere again depend on the nature of the field and, in particular, on the way in which the field varies over each tesseral element. Hence it is convenient to include errors due to this cause along with errors in the value of $\Omega+2aZ$.

(iii) The errors in the values of the basic integrals I_N^A are due almost entirely to rounding-off, i.e. they are less than one half of the last figure retained. Hence errors due to this cause are negligible compared with the errors in $\Omega+2aZ$.

(iv) The evaluation of the scalar product may be compared to the calculation of a weighted mean of the values of $\Omega+2aZ$, the values of I_N^A being the weighting factors whose sum is unity. Since the errors in $\Omega+2aZ$ may be expected to be nearly random, a certain amount of cancellation of error is probable except in unfavourable circumstances, e.g. when large values of I_N^A are multiplied by values of $\Omega+2aZ$ in which the errors are large and of the same sign. The cumulative effect of the rounding errors of the products is made negligible by the retention of a sufficient number of extra figures.

An examination of the sources of error suggests that the errors in the calculation of Ω_e and Ω_i by this method are not likely to be as serious as those in the determination of the field elements Ω and Z. A test calculation has confirmed this.

3·3. *The calculation of the basic integrals for a sphere*

The basic integrals I_N^A defined in equation (3·7) could be evaluated by repeated integration, inserting the value of γ given by the expression.

$$\cos 2\gamma = \sin\theta\sin\theta_A\cos(\lambda-\lambda_A)+\cos\theta\cos\theta_A. \tag{3·10}$$

However, this does not provide a convenient method of calculation. It is also possible to expand $1/R_A$ as a series of Legendre polynomials, but this again proves unsatisfactory. It was found easiest to do the calculations by successive approximations; at the time that the calculations were done only a desk machine was available.

The first approximation to the value of the integral I_N^A is $\Delta S_N/(4\pi aR_A)$, where R_A is the distance from the point Q_N to the point A. The second approximation is obtained by dividing the tessera symmetrically into four parts and summing the similar expressions for each part; these parts may be again subdivided, and the process continued until the successive approximations to I_N^A are close enough to allow an accurate estimate of the limiting value.

The values of I_N^A are largest when Q_N is near A. Then the determination of these values is laborious, since $1/R_A$ varies considerably within each tessera. In particular it is necessary to calculate I_A^A, but in this case $R_A = 0$ and the first approximation given above is infinite; it is, however, possible to calculate the succeeding approximations. Moreover, the surface in the immediate neighbourhood of A is nearly plane, so that a small tessera centred at A may be treated as a plane trapezium, for which an accurate value of the integral can be found (see § 3·4). Calculations of the probable error and comparison with the other approximations indicate the absolute errors of the values of the integrals I_A^A are not likely to be excessive compared with those of the other integrals, and the proportionate errors are certainly comparable.

The values of I_N^A were calculated for tesserae bounded by small circles at intervals of 10° and meridians at intervals of 15°. The separate values of I_N^A were obtained for points Q_N at colatitudes $\theta_N = 0°, 10°, 20°, \ldots, 180°$ and longitudes $\lambda_N = 0°, 15°, 30°, \ldots, 180°$ and for points A at colatitude $\theta_N = 0°, 10°, \ldots, 90°$ and longitude $\lambda_A = 0°$. The other values may be obtained from considerations of symmetry.

The typical quantity to be calculated, namely $\Delta S_N/4\pi aR_A$, is of the form $q/(x+yz)^{\frac{1}{2}}$, where

$$\left.\begin{aligned} q &= (\beta/2\pi)\sin\alpha\sin\theta_N, & x &= \sin^2\tfrac{1}{2}(\theta_N-\theta_A),\\ y &= \sin\theta_N\sin\theta_A, & z &= \sin^2\tfrac{1}{2}\lambda_N. \end{aligned}\right\} \tag{3·11}$$

Here A is the point $(\theta_A, 0)$, Q_N is the point (θ_N, λ_N) and the tessera lies between colatitudes $\theta_N\pm\alpha$ and longitudes $\lambda_N\pm\beta$. The values of α, β, θ_N and λ_N depend on the subdivisions of the

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF

original tessera; q, x, y and z are common to several tesserae, but occur in different combinations; q, x and z occur frequently, but y occurs only for tesserae at the same latitude and for the same point A. These basic quantities are evaluated first, and then $q/(x+yz)^{\frac{1}{2}}$ can be calculated in such a way that only the result need be recorded.

The values of I_N^A are found to lie between 0·06 and 0·0003. Most of them lie between 0·010 and 0·001. It was decided to calculate the values to the fifth decimal place, so that most would contain three significant figures. The intermediate stages of the calculation were made to five or six significant figures. With this accuracy it was found that the first approximation was valid for more than half the surface of the sphere. It was necessary to proceed to the fourth approximation (i.e. a subdivision into 64 tesserae) only for a very small portion of the surface.

The final summations over the whole sphere show that the total error is less than one part in a thousand, and the average systematic error in each value is certainly less than 0·025 of the last figure retained. The error is consistently negative; this is to be expected, since the true values of I_N^A are greater (in general) than the approximate values. The values of I_N^A are given in table 1; the values for 0° and 180° refer to the polar caps.

The method may be adapted to analyze a field in more detail in an area, such as the equatorial regions, over which the field varies considerably and can be determined at a finer interval. However, a subsidiary table of basic integrals for the finer mesh is needed for the area concerned. The additional programming and calculation would be kept to the minimum by arbitrarily dividing the field into a main part that can be represented adequately at a wide interval and the remaining part that is non-zero only in the area to be treated at the finer interval. The analyses of these parts are carried out independently. If the area is sufficiently small to be treated as a plane the basic integrals given in table 2 may be used to analyze the remaining part. The results for the main part are interpolated to the finer interval and the results for the remaining part are merely added to the interpolated values.

3·4. *The application to a plane*

For a plane surface, the expression (3·1) reduces to

$$(\Omega_e - \Omega_i)_A = \int_S \frac{Z\,dS}{2\pi R_A}, \tag{3·12}$$

the integrand being independent of the potential itself. This result can be obtained immediately from equation (3·5) by taking the limit as $a \to \infty$. It can also be obtained by expressing the potential in terms of Bessel functions, with the use of cylindrical polar co-ordinates. The formula is obtained on the supposition that the surface S is of infinite extent, but it may be applied (as an approximation) to practical problems in which the field is effectively zero over all but a finite area of the plane. It may be applied also (as an approximation) if the field over a sphere is zero for all but a small proportion of the surface.

Cartesian co-ordinates are the most convenient co-ordinates to use when applying the method to a plane. Polar co-ordinates suffer from the disadvantage that the given field must be transformed to a new set of co-ordinates for each point at which the separation is to be made, the point being taken as the pole of the co-ordinate system.

Table 1. Basic integrals (I_N^A) for a sphere for a ($10° \times 1^h$)-mesh

Difference of longitude, $|\lambda_N - \lambda_A|$

In units of the fifth decimal

θ_A	θ_N	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h
	0°	1130												
10°	10	2189	1048	633	450	350	291	251	229	210	197	188	183	182
	20	744	690	584	488	414	360	321	293	272	257	247	242	240
	30	526	513	481	440	399	364	335	312	294	281	272	267	266
	40	452	446	429	405	379	355	333	315	301	290	282	278	276
	50	407	403	392	377	359	341	325	311	299	290	283	280	278
	60	372	370	362	351	338	325	312	301	291	284	278	275	274
	70	341	339	334	326	316	306	296	287	280	273	269	266	265
	80	312	310	306	301	293	285	278	271	264	259	256	254	253
	90	282	281	279	274	269	263	257	251	246	242	239	238	237
	100	253	252	250	247	243	238	234	229	226	222	220	219	218
	110	223	222	221	218	215	212	209	206	203	200	199	198	197
	120	192	192	191	189	187	184	182	180	178	176	175	174	174
	130	161	160	160	159	157	156	154	152	151	150	149	148	148
	140	129	129	128	128	127	126	125	124	123	122	121	121	121
	150	97	97	96	96	95	95	94	94	93	93	92	92	92
	160	64	64	64	64	64	64	63	63	63	63	62	62	62
	170	32	32	32	32	32	32	32	32	32	32	32	32	32
	180	96												
	0°	552												
20°	10	428	380	305	249	209	181	161	149	138	131	126	123	122
	20	3355	1298	687	470	361	297	256	229	210	197	188	183	182
	30	1090	908	662	504	405	341	297	267	246	232	222	217	215
	40	677	638	553	469	401	350	312	284	264	250	241	235	233
	50	539	522	479	429	383	344	313	289	271	258	249	244	243
	60	460	451	426	393	360	331	306	286	270	259	251	246	245
	70	404	398	381	359	335	313	293	276	263	254	247	243	241
	80	358	354	343	327	309	292	276	263	252	244	238	235	233
	90	317	314	306	295	282	269	257	246	237	230	226	223	222
	100	278	276	271	263	254	244	234	226	219	214	210	207	206
	110	241	240	236	231	224	217	210	204	198	194	191	189	188
	120	205	204	202	198	193	188	183	179	175	172	169	168	167
	130	170	169	168	165	162	159	155	152	149	147	145	144	144
	140	135	134	133	132	130	128	126	124	122	121	120	119	119
	150	100	100	99	99	98	97	95	94	93	92	92	91	91
	160	66	66	66	65	65	65	64	63	63	63	62	62	62
	170	33	33	33	33	32	32	32	32	32	32	32	32	32
	180	97												
	0°	370												
30°	10	193	186	172	154	139	126	116	108	102	98	95	93	92
	20	793	637	453	344	276	232	203	183	169	158	152	148	147
	30	4198	1394	702	474	363	298	257	229	210	197	188	183	182
	40	1388	1043	689	501	394	328	285	255	234	220	211	205	204
	50	805	730	592	475	393	336	296	267	247	233	223	218	216
	60	609	578	509	437	376	330	296	270	251	238	229	224	222
	70	499	483	443	396	353	317	288	266	249	237	229	224	223
	80	423	414	389	357	326	298	274	256	242	231	224	220	218
	90	363	357	341	319	297	275	257	242	230	221	215	211	210
	100	312	308	297	282	266	250	236	224	214	207	201	198	197
	110	265	263	256	246	234	223	212	203	195	189	185	183	182
	120	222	221	216	210	202	194	186	179	173	169	165	163	163
	130	182	181	178	173	168	163	158	153	149	146	143	142	141
	140	143	142	140	138	135	131	128	125	122	120	118	117	117
	150	105	105	104	102	101	99	97	95	94	93	92	91	91
	160	69	68	68	68	67	66	65	64	64	63	63	62	62
	170	34	34	33	33	33	33	33	33	32	32	32	32	32
	180	99												

Table 1 (*continued*). Basic integrals (I_N^A) for a sphere

Difference of longitude, $|\lambda_N - \lambda_A|$

In units of the fifth decimal

θ_A	θ_N	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
	0°	279												
40°	10	126	124	118	111	103	96	90	85	81	78	76	75	75
	20	370	346	297	250	213	186	166	151	141	133	128	125	124
	30	1121	820	535	389	306	255	221	198	182	171	164	160	158
	40	4790	1439	708	476	364	299	257	229	210	197	188	183	182
	50	1629	1126	697	496	386	320	276	247	227	213	204	198	197
	60	908	794	609	472	383	324	284	255	235	221	212	207	205
	70	660	615	522	434	366	317	281	255	237	224	215	210	208
	80	523	501	449	391	341	302	272	249	233	221	213	208	206
	90	430	417	386	348	312	282	257	238	223	213	206	202	200
	100	358	350	331	306	280	257	238	222	210	201	195	192	190
	110	298	293	281	264	246	230	215	203	193	186	181	178	177
	120	245	242	234	224	212	200	189	180	173	167	163	161	160
	130	197	195	191	184	176	168	161	155	149	145	142	140	140
	140	152	151	149	145	141	136	131	127	123	120	118	117	117
	150	111	110	109	107	105	102	100	97	95	93	92	91	91
	160	72	72	71	70	69	68	67	66	65	64	64	63	63
	170	35	35	35	34	34	34	34	33	33	33	33	33	33
	180	101												
	0°	226												
50°	10	94	93	90	86	82	77	73	70	68	66	64	63	63
	20	245	236	216	192	171	153	139	129	121	115	111	109	108
	30	533	482	387	310	256	219	193	174	161	152	146	142	141
	40	1403	949	584	415	323	268	232	207	190	178	171	167	165
	50	5223	1462	710	477	364	299	257	229	210	197	188	183	182
	60	1818	1174	697	489	379	313	270	241	221	208	199	194	192
	70	983	834	613	465	373	314	274	246	226	213	204	199	197
	80	691	634	524	426	355	304	269	243	225	212	204	199	197
	90	531	504	443	380	328	287	257	235	218	207	199	195	193
	100	423	409	374	333	295	264	240	221	207	197	190	186	185
	110	341	333	313	286	260	237	219	204	192	184	178	174	173
	120	274	270	257	241	224	208	194	182	173	166	162	159	158
	130	216	214	207	197	186	175	166	157	151	146	142	140	139
	140	165	164	160	154	148	141	135	130	125	122	119	118	117
	150	119	118	116	113	110	106	103	100	97	95	94	93	92
	160	76	76	75	74	72	71	69	68	67	66	65	64	64
	170	36	36	36	36	36	35	35	35	34	34	34	34	34
	180	105												
	0°	191												
60°	10	75	74	73	71	68	65	63	61	59	57	56	55	55
	20	184	179	169	156	142	131	121	113	107	102	99	97	97
	30	355	336	295	252	217	191	171	156	145	137	132	129	128
	40	682	593	452	350	284	240	210	189	175	164	158	154	152
	50	1635	1041	616	432	335	277	239	213	196	184	176	171	170
	60	5534	1476	712	478	364	299	257	229	210	197	188	183	182
	70	1952	1200	693	482	373	308	265	237	217	204	195	190	188
	80	1029	852	610	456	364	305	265	238	219	205	197	192	190
	90	701	637	516	414	342	292	257	232	215	202	194	190	188
	100	523	494	430	365	312	272	243	221	206	195	187	183	182
	110	404	389	354	313	276	246	223	205	192	183	176	173	171
	120	315	307	287	262	238	216	199	185	175	167	161	158	157
	130	243	239	228	213	198	183	171	161	153	147	143	141	140
	140	182	180	174	166	157	148	140	134	128	124	121	119	119
	150	128	128	125	121	117	112	107	103	100	97	95	94	94
	160	81	81	80	78	76	74	72	71	69	68	67	66	66
	170	39	38	38	38	37	37	37	36	36	35	35	35	35
	180	110												

Table 1 (*continued*). Basic integrals (I_N^A) for a sphere

Difference of longitude, $|\lambda_N - \lambda_A|$

In units of the fifth decimal

θ_A	θ_N	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h
	0°	166												
70°	10	63	63	62	60	59	55	55	53	52	51	50	49	49
	20	147	145	139	131	122	114	107	101	96	92	90	88	88
	30	267	258	237	211	188	168	153	141	133	126	122	119	119
	40	455	423	358	297	250	217	192	175	162	153	147	144	143
	50	808	682	500	379	304	256	223	200	184	173	166	162	161
	60	1816	1106	638	444	343	283	245	218	200	188	180	175	174
	70	5742	1483	713	478	364	299	257	229	210	197	188	183	182
	80	2029	1208	686	476	367	303	261	233	214	200	192	187	185
	90	1044	857	599	445	354	300	257	230	212	199	191	186	184
	100	691	623	500	399	328	280	246	222	205	193	185	181	180
	110	499	470	408	345	294	256	229	208	193	183	176	172	171
	120	372	363	326	288	255	227	206	189	177	168	162	159	158
	130	278	272	255	233	212	194	178	166	157	150	145	142	141
	140	204	200	192	181	169	157	147	139	132	127	124	122	121
	150	141	140	136	131	125	119	113	108	104	101	98	97	97
	160	88	87	86	84	82	79	76	74	72	71	70	69	69
	170	41	41	41	40	40	39	39	38	38	37	37	36	36
	180	116												
	0°	148												
80°	10	55	55	54	53	52	50	49	48	47	46	45	45	45
	20	124	123	119	114	108	102	96	91	88	85	83	82	81
	30	215	210	197	181	165	151	139	130	123	117	114	112	111
	40	343	328	293	255	223	197	177	163	152	144	139	136	135
	50	540	495	407	331	276	237	209	189	175	165	159	155	154
	60	909	750	536	401	320	268	233	209	192	181	173	169	167
	70	1946	1153	655	454	350	289	249	222	204	191	183	178	177
	80	5865	1487	713	478	365	299	257	229	210	197	188	183	182
	90	2055	1202	678	469	362	298	257	229	210	197	189	184	182
	100	1028	834	584	433	344	287	250	224	206	193	185	180	179
	110	660	595	477	381	313	267	235	212	196	184	177	173	171
	120	460	434	378	320	274	239	214	195	181	171	165	161	160
	130	329	318	291	259	230	206	187	172	161	153	148	145	144
	140	234	229	216	199	183	168	155	145	137	131	127	125	124
	150	158	156	151	143	135	127	120	114	109	105	102	101	100
	160	97	96	94	91	88	85	82	79	76	74	73	72	72
	170	45	45	44	44	43	42	41	41	40	39	39	39	39
	180	124												
90°	0°	135												
	10	49	49	48	48	47	46	45	44	43	42	42	41	41
	20	108	107	105	101	97	92	88	84	81	79	77	76	76
	30	182	179	171	160	148	138	128	121	115	110	107	106	105
	40	277	269	249	224	201	181	165	153	144	137	132	130	129
	50	408	387	340	291	251	220	197	180	167	159	153	149	148
	60	609	552	447	358	297	253	223	201	186	175	168	164	163
	70	983	801	563	418	332	278	242	217	199	187	179	175	173
	80	2025	1184	668	462	356	294	254	226	207	194	186	181	180
	90	5905	1488	713	483	364	299	257	229	210	197	188	183	182

The values for $(180° - \theta_N)$ and θ_N are equal.

θ_A	θ_N	I_N^A	θ_N	I_N^A	θ_N	I_N^A	θ_N	I_N^A
0°	0°	4362	50°	329	100°	234	150°	94
	10	362	60	315	110	208	160	63
	20	358	70	298	120	182	170	32
	30	351	80	278	130	154	180	95
	40	342	90	257	140	124		

I_N^A is independent of λ_N.

In using Cartesian co-ordinates the region in which the field is non-zero is divided into a large number of equal rectangles ΔS_N, having centres Q_N. Then, as in the case of the sphere, the integral is expressed in the approximate form

$$(\Omega_e - \Omega_i)_A \simeq \sum_N Z_N \int_{\Delta S_N} \frac{dS}{2\pi R_A} = \sum_N Z_N I_N^A. \tag{3·13}$$

Here Z_N is the value of the vertical component at Q_N, or its mean value over the rectangle ΔS_N. The basic integrals I_N^A may be evaluated rigorously, and it is only necessary to calculate one set of integrals for a given mesh, since the mesh-points are unchanged when the origin is changed from one mesh-point to another. The evaluation of the scalar products involves techniques similar to those for a sphere (§ 8·1) but they are less elaborate.

In evaluating the basic integrals for this case it is convenient to take A as the origin and the sides of the rectangle ΔS_N as the lines

$$x = x_1, \quad x = x_2; \quad y = y_1, \quad y = y_2. \tag{3·14}$$

We have then

$$2\pi I_N^A = \int_{y_1}^{y_2}\int_{x_1}^{x_2} (x^2+y^2)^{-\frac{1}{2}}\, dx\, dy = L(x_2, y_2) - L(x_2, y_1) - L(x_1, y_2) + L(x_1, y_1), \tag{3·15}$$

where

$$L(x, y) = x \ln\{y + (x^2+y^2)^{\frac{1}{2}}\} + y \ln\{x + (x^2+y^2)^{\frac{1}{2}}\}. \tag{3·16}$$

In general a square mesh is likely to be most useful. In this case the point A is assumed to be at the centre of a square. If the side of the square is taken to be $2a$, the values of x_1, x_2, y_1, y_2 are of the form

$$x_1 = (2n-1)\,a, \quad x_2 = (2n+1)\,a; \quad y_1 = (2m-1)\,a, \quad y_2 = (2m+1)\,a. \tag{3·17}$$

Here n and m are integers. Reference to any square may be made by its 'co-ordinates' (n, m). It is convenient to define λ, μ, ν and ρ by the relations

$$R_{11} = \lambda a, \quad R_{21} = \mu a, \quad R_{22} = \nu a, \quad R_{12} = \rho a.$$

Here R_{ij} is the distance from the point (x_i, y_j) to the point A. Then

$$\frac{2\pi}{a} I_N^A = (2n+1) \ln \frac{2m+1+\nu}{2m-1+\mu} + (2m+1) \ln \frac{2n+1+\nu}{2n-1+\rho}$$
$$- (2n-1) \ln \frac{2m+1+\rho}{2m-1+\lambda} - (2m-1) \ln \frac{2n+1+\mu}{2n-1+\lambda}. \tag{3·18}$$

From the considerations of symmetry it is clearly only necessary to consider the cases $n = 0, 1, 2, \ldots$, and $0 \leqslant m \leqslant n$.

If either n or m is zero equation (3·18) simplifies, and if both are zero we have

$$(2\pi/a)\, I_N^A = 8 \ln(\sqrt{2}+1). \tag{3·19}$$

If n or m is sufficiently large it is possible with sufficient accuracy to substitute the approximation $\Delta S_N/(2\pi R_A)$ for the basic integral. Here R_A is the distance from Q_N to the origin A. Hence

$$(2\pi/a)\, I_N^A \simeq 2(n^2+m^2)^{\frac{1}{2}}. \tag{3·20}$$

This formula is accurate to one part in a thousand when $(n^2+m^2)^{\frac{1}{2}}$ is greater than 7, and to one part in a hundred when it is just greater than 2, so that the effects of the variation of $(1/R)$ over a square are quite small except near the origin A.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Table 2 gives the values of $(2\pi/a)\, I_{\mathrm{N}}^{\mathrm{A}}$ to three decimals for $n = 0, 1, \ldots, 10$, and $0 \leqslant m \leqslant n$.

The value of the basic integral can also be obtained in terms of logarithmic functions when ΔS is any plane polygon. The special case of a symmetrical trapezium centred at the origin (e.g. PQRS, where PQ > RS and QR = SP) is of interest because of its application to the approximate evaluation of $I_{\mathrm{A}}^{\mathrm{A}}$ for a sphere (see § 3·3). If PQ = $2b$, RS = $2a$ and the height of the trapezium is $2c$, it is found that

$$\int_{\Delta S} \frac{\mathrm{d}S}{R} = 2c\left\{\ln\frac{x+a}{c} + \ln\frac{y+b}{c} + \frac{d}{z}\ln\frac{yz+be+c^2}{xz+ae-c^2}\right\}, \tag{3·21}$$

where
$$d = \tfrac{1}{2}(a+b), \quad e = \tfrac{1}{2}(b-a),$$
$$x = (a^2+c^2)^{\frac{1}{2}}, \quad y = (b^2+c^2)^{\frac{1}{2}}, \quad z = (e^2+c^2)^{\frac{1}{2}}.$$

Table 2. Basic integrals for a plane square mesh

The tabulated function is $(2\pi/a)\, I_{\mathrm{N}}^{\mathrm{A}}$, where $I_{\mathrm{N}}^{\mathrm{A}}$ is defined by equation (3·13) and the mesh-squares are each of side $2a$; A is the point (0, 0) and N is the point $(2na, 2ma)$.

n	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	
10	0·200	0·199	0·196	0·192	0·186	0·179	0·172	
9	0·222	0·221	0·217	0·221	0·203	0·194	0·185	
8	0·250	0·248	0·243	0·234	0·224	0·212	0·200	
7	0·286	0·283	0·275	0·263	0·248	0·233	0·217	
6	0·334	0·330	0·317	0·298	0·278	0·256	0·236	
5	0·401	0·393	0·372	0·343	0·313	0·283	——	
4	0·501	0·486	0·448	0·401	0·354	——		
3	0·670	0·635	0·556	0·473	——		0·202	**7**
2	1·010	0·902	0·711	——		0·177	0·188	**8**
1	2·076	1·449			0·157	0·166	0·176	**9**
0	7·051			0·141	0·149	0·156	0·164	**10**
				m = **10**	m = **9**	m = **8**	m = **7**	n

3·5. *The separation of the vertical component*

If the potential over the surface of a sphere has been separated into parts of external and internal origin it is possible to derive the corresponding contributions to the vertical component to any point A by using the surface-integral formula

$$(Z_{\mathrm{e}})_{\mathrm{A}} = -\int_S \frac{\{\Omega_{\mathrm{e}} - (\Omega_{\mathrm{e}})_{\mathrm{A}}\}}{2\pi R_{\mathrm{A}}^3}\,\mathrm{d}S, \tag{3·22}$$

together with the simple relation

$$(Z_{\mathrm{i}})_{\mathrm{A}} = Z_{\mathrm{A}} - (Z_{\mathrm{e}})_{\mathrm{A}}. \tag{3·23}$$

There are similar surface-integral formulae for Z_{i} (in terms of Ω_{i}) and for $Z_{\mathrm{e}} - Z_{\mathrm{i}}$ (in terms of Ω and Z), but the above pair of formulae appear to be the most suitable. The evaluation of Z_{i} directly from Ω_{i} would, however, provide a useful check on the numerical consistency of the results if sufficient computer time were available.

We can evaluate equation (3·22) by techniques similar to those used for the separation of the potential, but slight modifications are required, since the integral $I_{\mathrm{N}}^{\mathrm{A}}$ given by

$$I_{\mathrm{N}}^{\mathrm{A}} = \int_{\Delta S_{\mathrm{N}}} \frac{\mathrm{d}S}{2\pi R_{\mathrm{A}}^3} \tag{3·24}$$

has a singularity when ΔS_{N} is centred at A. However, since $\Omega_{\mathrm{e}} - (\Omega_{\mathrm{e}})_{\mathrm{A}}$ vanishes at A, we can evaluate

$$\int_{\Delta S_{\mathrm{N}}} \frac{\{\Omega_{\mathrm{e}} - (\Omega_{\mathrm{e}})_{\mathrm{A}}\}}{2\pi R_{\mathrm{A}}^3}\,\mathrm{d}S \tag{3·25}$$

to sufficient accuracy using simple approximations. The result can then be added to the scalar product

$$\sum_{N} \{\Omega_e - (\Omega_e)_A\} I_N^A, \tag{3·26}$$

in which I_N^A is as defined in equation (3·24) except that I_A^A is taken to be zero. A set of values of I_N^A can be computed once and for all using methods similar to those described in §3·3. No attempt has yet been made, however, to separate the vertical force by this procedure.

When the field is given over a plane the formula for the separation of the vertical component is

$$(Z_e - Z_i)_A = -\int_S \frac{\Omega - \Omega_A}{2\pi R_A^3} \, dS; \tag{3·27}$$

the integral is independent of the vertical component itself. The integral can be evaluated by procedures similar to those described above.

An alternative formula can be obtained by differentiating with respect to z the expression

$$(\Omega_i)_A = \int_S \frac{z\Omega_i}{2\pi R_A^3} \, dS \tag{3·28}$$

for the potential due to sources in $z < 0$ at a point A (x, y, z) in the region $z > 0$. (Jeffreys & Jeffreys 1950, p. 221). Writing $dS = dx\,dy$, integrating by parts with respect to x and then making $z \to 0$ lead to the expression

$$(Z_i)_A = -\int_S \frac{(x - x_A)\, X_i(x, y)}{2\pi R_A (y - y_A)^2} \, dS. \tag{3·29}$$

The corresponding expression for $(Z_e)_A$ is similar but of opposite sign, so that on subtraction we have

$$(Z_e - Z_i)_A = \int_S \frac{(x - x_A)\, X(x, y)}{2\pi R_A (y - y_A)^2} \, dS. \tag{3·30}$$

There is of course a similar formula in terms of the Y-component. These formulae make it possible to separate the external and internal parts of the Z-components by using one of the horizontal components only instead of the potential Ω.

It is also worth noting that formulae can be found in the same way for separating each horizontal component into its two parts without introducing the potential Ω. We find in fact that

$$(X_e - X_i)_A = -\int_S \frac{(x - x_A)\, Z(x, y)}{2\pi R_A^3} \, dS, \tag{3·31}$$

and there is a corresponding formula for $(Y_e - Y_i)_A$.

The above formulae have been derived on the assumption that the field tends to zero as x or y become infinite on the plane, and are not therefore applicable to the case of a two-dimensional field depending on x and z only. But in this case elementary potential theory leads to the formula

$$(X_e - X_i)_A = -\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{Z(x)}{(x - x_A)} \, dx, \tag{3·32}$$

and a similar formula for $(Z_e - Z_i)_A$.

A method of separating a surface magnetic field given over a plane into parts of external and internal origin has also been described by Kertz (1954) and Siebert & Kertz (1957). The formulae they derive are effectively the same as some of those above, but it is believed that the present methods of derivation and application of these formulae may be somewhat simpler and more convenient.

II. THE ANALYSIS OF THE Sq FIELD OF 1932–3

4. The selection and reduction of the data

4·1. *Introduction*

The methods described in part I have been applied to the values of the quiet-day variations (Sq) obtained during the Second International Polar Year, which extended from August 1932 to August 1933 inclusive. Also some later observations have been examined, in order to gain a closer knowledge of particular features of the Sq field, and thus assist in the analysis of the Polar Year data and their interpolation over the earth. Reference has already been made in part I to several previous studies of the Polar Year data, and to some of the difficulties and disadvantages of the methods used in these studies.

The months of the Polar Year were grouped in the usual manner into the three 'seasons': (E) the equinoctial months, September, October 1932 and March, April 1933; (D) the December solstitial months, November, December 1932, and January, February 1933: (J) the June solstitial months, May, June, July and August 1933. The averages for the 20 international quiet days of each season of the daily variations of the three magnetic elements X, Y, Z at each observatory, corrected for non-cyclic change, were first determined. The actual interpolation of these variations over the earth and the determination of the potential of the Sq field have been made for the J-months only, but the observatory data have been reduced to the form necessary for extending the analysis to the other seasons. The J-months were chosen because the data for these months are more complete and reliable than for the D- or E-months.

In none of the earlier analyses was it possible to take adequate account of the great enhancement of Sq near the magnetic equator when deriving the potential to represent the surface field. The remarkably large variations of the horizontal component H, first noticed at Huancayo (Johnston & McNish 1932) are now known to occur in all those regions near the magnetic equator where observations have so far been made. They are generally regarded as due to a considerable concentration of the ionospheric current-system near the equator, now known as the equatorial electrojet. Some recent writers have tended to regard the field of the electrojet as an *addition* to the 'normal Sq field'. However, while we realise the importance of isolating certain features of the field for special study, we here regard the large equatorial variations as an essential feature of the Sq field itself, and in our analysis we have attempted to take some account of the admittedly scant data (for 1932–3) about them.

It is difficult to determine the true Sq field in latitudes greater than about 60° N because in these high latitudes even the seasonal mean variations for the 5 international quiet days of each month are still considerably affected by disturbance. Hasegawa (1940) found that the daily variations in these high latitudes derived from observations on the international quiet days of the Polar Year resembled the SD variations, and Nagata & Mizuno (1955) have pointed out that these days were by no means free from disturbance; the mean value of the daily sum of the K_p-indices for them was 5·1. Nagata & Mizuno have shown, however, that on the very quietest days ($\Sigma K_p < 2$) the variations are similar to those which would be obtained by a simple extrapolation of the Sq field from lower latitudes. Hence to avoid as far as possible introducing extraneous features we have used only the data from stations south of about 60° N in the main part of our investigations; there were no observations made in Antarctica that were available for use.

Our method of analysis differs considerably from that used in earlier investigations. Instead of analyzing the geographical distribution of the Fourier coefficients of the daily variations at the observatories, we have attempted to determine the instantaneous distribution of the actual field at a number of selected epochs. The epochs selected were 4^h, 8^h, 12^h and 16^h U.T. The observatories in the Pacific Ocean area were too few to enable satisfactory estimates to be made of the distribution at 0^h and 20^h U.T., because at these times the major variations of the field would be situated within this area. However, during the 12^h from 4^h to 16^h U.T. the local-noon meridian crosses the region where there are the greatest variations in the position of the geomagnetic equator relative to the geographic equator, and where there are considerable changes in the distribution of land and sea. This period may therefore be expected to show the most important changes in the Sq field as the earth rotates. However, a determination of the field when the main current systems were over the Pacific Ocean would be of considerable interest since the effects of induction in the oceans could be evaluated more easily than at other times.

To obtain a true picture of the average Sq field, it is necessary to eliminate as far as possible the effects of disturbance, non-cyclic changes and day-to-day variability. A closely associated problem is the choice of the datum line from which to measure the daily variations. Our examination of the data suggests that the variation at each observatory should be expressed as a deviation from a suitably chosen night value, rather than from the usual daily-mean value. This choice of datum line causes our results to differ somewhat from those of earlier analyses.

4·2. *The observatories*

Records of the daily variations and other relevant data were obtained for 59 observatories operating during the Polar Year. Of these 44 were situated between geographic latitudes 60·1° N and 60·8° S (geomagnetic latitudes 62·5° N and 50° S). The main analysis was based on observations from this group. The remaining 15 were in or near the northern auroral zone. Some observations for a limited period from one other station (Tatuoca) were also used. The geographic longitudes (λ) and latitudes (ϕ) of the observatories are given in table 3; it includes abbreviations for the observatory names, which are used elsewhere in our work. The geographical distribution is shown later in figure 7.

Table 3 also includes the magnetic latitudes Φ, Θ, M for each observatory; these are defined as follows. The geomagnetic latitude Φ is referred to the axis of the centred dipole with the geomagnetic north pole at $\lambda = 69°$ E, $\phi = 78·5°$ N. The local magnetic latitude Θ is defined by $\tan\Theta = \frac{1}{2}\tan I$, where I is the magnetic inclination. The mean magnetic latitude M is defined by $\mathrm{M} = \frac{1}{2}(\Phi + \Theta)$; it has been introduced here since some features of the Sq field appear to depend more closely on M than on either Φ or Θ.

The records of a further 13 observatories were inspected, but were judged to be insufficiently complete or otherwise unsuitable for the present investigation. These observatories were Bossekop, Coimbra, Dombås, Jan Mayen, Kandalakcha, Ksara, San Fernando, San Miguel, Swider, Tromsö, Vyssokaya Doubrawa and Yakutsk. No records were obtainable for a further 5 observatories which are believed to have operated during the Polar Year. These were Dehra Dun, Karsani, Mai-Toon, Tashkent and Tsingtau.

TABLE 3. CO-ORDINATES OF THE 1932–3 POLAR YEAR OBSERVATORIES

Notation

λ = longitude east of Greenwich, (1^h = 15°).
ϕ = geographic latitude.
Φ = geomagnetic latitude.
Θ = local magnetic latitude.
M = mean magnetic latitude = $\frac{1}{2}(\Phi+\Theta)$.

	observatory		longitude λ	north latitude ϕ	Φ	Θ	M
			h	°	°	°	°
1	Le	Lerwick	−0·08	+60·1	+62·5	+59·0	+60·8
2	Sl	Sloutsk	+2·03	+59·7	+56·0	+56·8	+56·4
3	Lo	Lovö	+1·19	+59·4	+58·1	+56·3	+57·2
4	Si	Sitka	+14·98	+57·0	+60·0	+60·7	+60·3
5	RS	Rude Skov	+0·83	+55·8	+55·8	+53·2	+54·5
6	Za	Zaimische	+3·26	+55·8	+49·3	+55·0	+52·2
7	Es	Eskdalemuir	−0·21	+55·3	+58·5	+53·6	+56·0
8	Me	Meanook	+16·44	+54·6	+61·8	+66·8	+64·3
9	Zo	Zouy	+6·94	+52·5	+41·0	+56·0	+48·5
10	DB	De Bilt	+0·33	+52·1	+53·8	+49·7	+51·8
11	Ab	Abinger	−0·02	+51·2	+54·0	+49·2	+51·6
12	Ma	Manhay	+0·38	+50·3	+52·0	+48·0	+50·0
13	VJ	Val Joyeux	+0·13	+48·8	+51·3	+46·7	+49·0
14	Ty	Toyohara	+9·52	+47·0	+36·9	+41·7	+39·3
15	Ag	Agincourt	+18·72	+43·8	+55·0	+61·5	+58·2
16	Eb	Ebro	+0·03	+40·8	+43·9	+38·0	+41·0
17	Ch	Cheltenham	+18·88	+38·7	+50·1	+55·8	+53·0
18	Ka	Kakioka	+9·35	+36·2	+26·0	+30·3	+28·2
19	Tu	Tucson	+16·61	+32·2	+40·4	+40·5	+40·5
20	Lu	Lukiapang	+8·07	+31·3	+20·0	+27·0	+23·5
21	He	Helwan	+2·09	+29·9	+27·2	+24·1	+25·6
22	AT	Au Tau	+7·60	+22·4	+11·0	+16·0	+13·5
23	Ho	Honolulu	+13·46	+21·3	+21·1	+22·2	+21·6
24	Te	Teoloyucan	+17·39	+19·8	+29·6	+28·2	+28·9
25	Al	Alibag	+4·86	+18·6	+9·5	+13·4	+11·5
26	SJ	San Juan	+19·59	+18·4	+29·9	+33·1	+31·5
27	An	Antipolo	+8·08	+14·6	+3·3	+8·1	+5·7
28	FP	Fernando Po	+0·58	+3·4	+5·7	−7·4	−0·8
29	Mo	Mogadiscio	+3·02	+2·0	−2·7	−8·5	−5·6
30	Ba	Batavia	+7·12	−6·2	−17·6	−17·5	−17·5
31	El	Elizabethville	+1·83	−11·7	−12·7	−27·3	−20·0
32	Hu	Huancayo	+18·98	−12·0	−0·6	+1·0	+0·2
33	Ap	Apia	+12·55	−13·8	−16·0	−16·3	−16·1
34	Tn	Tananarive	+3·17	−18·9	−23·7	−35·0	−29·4
35	Mr	Mauritius	+3·84	−20·1	−26·6	−33·0	−29·8
36	LQ	La Quiaca	+19·63	−22·1	−10·6	−6·2	−8·4
37	Va	Vassouras	+21·09	−22·4	−11·9	−8·9	−10·4
38	Wa	Watheroo	+7·72	−30·3	−41·8	−46·1	−44·0
39	Pi	Pilar	+19·74	−31·7	−20·2	−13·7	−17·0
40	Ca	Capetown	+1·23	−33·9	−32·7	−44·6	−38·6
41	To	Toolangi	+9·70	−37·5	−46·7	−50·8	−48·7
42	Am	Amberley	+11·52	−43·2	−47·7	−52·1	−49·9
43	Mg	Magallanes	+19·27	−53·2	−41·6	−29·9	−35·8
44	OS	Orcadas del Sud	+21·01	−60·7	−50·0	−35·5	−42·8
	Ta	Tatuoca	+20·77	−1·2	+9·6	+11·4	+10·5

Table 3 (*continued*)

observatory			longitude λ	ϕ	north latitude Φ	Θ	M
			h	°	°	°	°
A 1	CB	Calm Bay	+ 3·52	+80·3	+71·5	+76·4	+74·0
A 2	Sv	Sveagruvan	+ 1·12	+77·9	+73·9	+72·4	+73·2
A 3	Th	Thule	+19·41	+76·5	+88·0	+80·7	+84·4
A 4	BI	Bear Island	+ 1·28	+74·5	+71·1	+69·8	+70·5
A 5	Di	Dickson	+ 5·36	+73·5	+63·0	+76·3	+69·6
A 6	MS	Matotchkin Shar	+ 3·76	+73·6	+64·8	+71·5	+68·2
A 7	SS	Scoresby Sund	+22·54	+70·5	+75·8	+67·5	+71·6
A 8	Pe	Petsamo	+ 2·08	+69·5	+64·9	+65·9	+65·4
A 9	Go	Godhavn	+20·43	+69·2	+79·8	+73·5	+76·7
A 10	So	Sodankyla	+ 1·78	+67·4	+63·8	+63·8	+63·8
A 11	Ak	Angmagssalik	+21·49	+65·6	+74·2	+67·5	+70·8
A 12	CF	College-Fairbanks	+14·14	+64·9	+64·5	+65·7	+65·1
A 13	CI	Chesterfield Inlet	+17·95	+63·3	+73·5	+82·8	+78·2
A 14	FR	Fort Rae	+16·26	+62·8	+69·0	+75·6	+72·3
A 15	Ju	Juliannehaab	+20·93	+60·7	+70·8	+66·3	+68·5

Table 4. A comparison of the distributions of observatories used in analyses of the Sq field

	S.C.	A.M.	M.H.	H. & O.	N.B.	P. & W.
total number of observatories	21	5	36*	53	46	44†
North polar region 60° N < ϕ	0	0	0	7	8	0‡
European regions 35° N < ϕ < 60° N 10° W < λ < 30° E	5	0	12	13	9	10
Equatorial regions 15° S < ϕ < 15° N 15° S < Φ < 15° N	4	1	3	6	6	11†
Southern hemisphere ϕ < 0° N	6	2	6	12	9	15†

Notes

author		dates of observations
S.C.	= S. Chapman 1919	1902, 1905
A.M.	= A. G. McNish 1937	1923
M.H.	= M. Hasegawa 1936	1933, 1934
H. & O.	= M. Hasegawa & M. Ota 1950	1932–3
N.B.	= N. P. Benkova 1940	1933
P. & W.	= Present analysis	1932–3

ϕ = geographic latitude
λ = geographic longitude
Φ = geomagnetic latitude

* An additional three observatories were used in part of the analysis.
† Tatuoca is not included in these totals.
‡ The data for fifteen auroral-zone observatories were examined, but it was found that they could not be used in the analysis.

A comparison with previous investigations is given in table 4. Apart from data for two Siberian observatories used by Benkova (1940), the previous analyses did not use any observations in areas not covered by the present one, and there were far fewer observations from equatorial regions. With regard to the data from the north polar regions, these were collected and reduced for further investigation, but were not used in the present analysis for the reason given in § 4·1.

4·3. *The data*

4·31. *The forms and sources of the data*

Our first objective was to obtain as complete a picture as possible of the Sq field over the earth at a number of epochs, universal time. This cannot be done for a particular epoch from a knowledge of the field at each observatory at that epoch *only*, because the observations are not sufficiently numerous or well placed. It is necessary to use a good deal of interpolation, based on a qualitative knowledge of the general features of the field and on some reasonable assumptions about it, e.g. that it possesses a potential. In most of the previous investigations the interpolation was done more or less automatically by assuming the field expressible in terms of a limited number of spherical harmonics. We have used a new method of interpolation, for which a preliminary requirement is a knowledge of the seasonal mean daily variations of the north (X), east (Y) and downward vertical (Z) field components referred to the exact hours of local mean time at each observatory.

To obtain this knowledge some processing of the original observatory data was necessary. The amount of work involved for each observatory depended on the form of the published tables or other records. These fell into three main classes, which in order of their suitability for our purpose were:

(i) Tables giving values of the monthly mean hourly departures ΔX, ΔY, ΔZ derived from the 5 international quiet days, which start at 0^h U.T. These departures were usually corrected for non-cyclic change, and were usually measured from the daily mean.

(ii) Tables giving the monthly mean hourly values of H, D, V or, in some cases, the monthly mean hourly departures ΔH, ΔD, ΔV for the international quiet days or for the corresponding days of local time. These hourly departures were rarely corrected for non-cylic change.

(iii) Tables giving the hourly values of H, D, V on all days. In this case the means for the quiet days had to be calculated. The non-cyclic change was sometimes given explicitly, but usually had to be determined.

In one case (Fernando Po) microfilm copies of the original daily traces of the elements were obtained, and the hourly values determined from these.

The published hourly values were sometimes instantaneous values at the exact hours, but more generally were mean values over consecutive hourly intervals centred on the hour or on the half-hour. Since an average of 20 days was used, it was not considered necessary to distinguish between instantaneous and mean values.

The sources from which the observatory data were obtained included observatory year-books, special Polar-Year publications, microfilm or photostat copies of observatory manuscripts and microfilms of the actual magnetograms. A detailed analysis of the published form and degree of completeness of the records is given by Wilkins (1951, table 21).

4·32. *The reduction of the data*

The five principal stages in the reduction of the data to the required form, when they are originally given in the form (iii) above, are as follows: (*a*) the calculation of the mean daily variations for each season; (*b*) the correction for non-cyclic change; (*c*) the conversion from H and D to X and Y; (*d*) the interpolation to local mean time (L.T.); and (*e*) the adjustment of the hourly values to measure them from a chosen base-line. Most of the observatories publish their data in form (i) or (ii) above, so that it was necessary in only a few cases to do all the calculations involved in stages (*a*), (*b*), and (*c*). Those in (*d*) and (*e*) were required in all cases.

Stage (*d*) is necessary because our method of determining the Sq field over the earth at selected epochs of universal time requires a knowledge of how the field, regarded as stationary relative to the sun, varies in intensity and distribution as the earth rotates. This in turn requires the daily variations at each station to be expressed as deviations at the exact hours of local mean time. In the case of the J-months these local-time values were determined by numerical interpolation using Bessel's formula. Graphical methods were found to be sufficiently accurate, and were used for the E- and D-months.

Stage (*e*) was necessary because, for reasons which we discuss later in § 5·52, we wished to represent the Sq field by the departures from a night value and not from the mean of the day.

The reduced data for the main group of 44 observatories are given in tables 5 to 13. These give the mean values of the north (X), east (Y) and downward vertical (Z) components of Sq at the exact hours of local mean time (L.T.) for each of the three seasons of the Polar Year. The values are expressed as deviations from a night value (see § 5·5) and the daily mean (M) and daily range (R) of these values are given. The values are corrected for the non-cyclic change (N) during the day. The unit is 1γ throughout. Partially reduced data for the northern auroral-zone observatories have also been tabulated by Wilkins (1951); stages (*d*) and (*e*) were not carried out as the best method of using these data was uncertain.

4·33. *The reliability of the reduced data*

The reliability of the values of the Sq variations used in the analysis depends on the accuracy and completeness of the original observations, and on the correctness of the methods of reduction. The observations were sometimes incomplete owing to instrumental faults or other failures; the loss of record ranged from a few hours to several days. Some of the temporary observatories did not operate during part of the Polar Year. When only occasional hourly values were missing, interpolated values were substituted. If a whole day's record was lost, the mean value for the month was calculated from less than the 5 international quiet days, or another quiet day was substituted. In obtaining the seasonal mean variations slightly affected months were given equal weight with complete months; in other cases weighted means were taken. In the rare cases when the whole month's observations were missing the seasonal mean was estimated either by using less than the 4 months, or by substituting data for the corresponding period in the preceding or succeeding year. Actually the losses for the J-months were slight or non-existent for all but two of the observatories in low and middle latitudes, but were more marked for the other seasons. Hence the data for the J-months have been used in the main investigation.

The accuracy of the original observations was in some cases difficult to estimate. The greatest instrumental accuracy would be expected at the permanent observatories, the errors of measurement being of the order of 1γ. This degree of accuracy was probably also achieved at many of the temporary observatories, but in a few cases internal evidence

TABLE 5. THE NORTH COMPONENT OF Sq, E-MONTHS 1932–3

unit = 1γ

L.T. =	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	*M*	*N*	*R*
1 Le	+1	0	−1	0	0	+1	0	−2	−8	−15	−22	−25	−24	−20	−13	−7	−2	0	+2	+3	+4	+3	+3	+2	−5	+4	29
2 Sl	+2	+1	0	−1	−1	−1	−3	−4	−6	−13	−20	−25	−23	−17	−10	−6	−3	−2	−1	0	+2	+3	+4	+3	−5	+2	29
3 Lo	+1	+1	+1	−2	−1	−1	−1	−3	−8	−14	−21	−24	−21	−16	−11	−7	−4	−2	0	+1	+2	+3	+2	+2	−5	+4	27
4 Si	0	0	+2	+2	+1	0	−3	−7	−13	−19	−21	−19	−14	−9	−4	−1	+1	+3	+2	0	−1	−1	0	0	−4	+0	24
5 RS	+2	+1	−1	−1	−1	−1	−1	−2	−6	−13	−20	−24	−22	−17	−12	−7	−4	−2	−1	+1	+2	+2	+2	+2	−5	+3	26
6 Za	+1	0	+1	0	0	+1	0	−1	−4	−8	−12	−14	−15	−13	−7	−2	−1	−1	0	0	+1	+2	+2	+1	−3	+1	17
7 Es	+2	+1	0	−1	−1	0	0	−2	−6	−13	−21	−26	−24	−20	−14	−8	−3	−1	+1	+3	+3	+2	+2	+3	−5	+4	29
8 Me	0	−1	−1	+1	+2	−1	−5	−4	−10	−16	−22	−22	−16	−9	−3	0	+3	+3	+3	+2	+1	0	0	+1	−4	+3	25
9 Zo	0	+1	0	0	−1	−2	−2	−4	−8	−15	−20	−21	−17	−9	−1	+4	+4	+1	0	−1	−2	−1	0	0	−4	+4	25
10 DB	+3	+1	−1	−2	−1	−1	0	0	−3	−10	−18	−22	−21	−17	−12	−6	−3	−1	+1	+2	+4	+4	+3	+3	−4	+2	26
11 Ab	+2	+1	−1	−2	−2	−1	−1	−2	−4	−10	−17	−21	−21	−17	−13	−9	−5	−3	−1	0	+1	+1	+1	+2	−5	+2	23
12 Ma	+2	−2	−3	−3	−2	0	0	−1	−2	−5	−11	−13	−14	−12	−10	−7	−5	−4	−2	−2	−1	−1	0	+2	−4	+2	16
13 VJ	+1	−1	−3	−4	−4	−3	−1	−2	−4	−9	−15	−17	−17	−15	−12	−9	−6	−4	−2	0	+1	+1	+1	+1	−5	+2	18
14 Ty	0	0	0	0	0	0	−1	0	−3	−10	−17	−19	−15	−10	−5	−1	−1	−1	−1	−3	−4	−2	−2	−1	−4	+3	19
15 Ag	0	−1	−1	0	+1	+1	+1	−2	−9	−19	−26	−28	−23	−15	−8	−0	+2	+2	+3	+1	0	0	0	0	−5	+4	31
16 Eb	+3	0	−1	−1	−1	−1	0	0	−1	−3	−3	−1	+1	+2	+1	−1	−3	−2	0	+1	+2	+3	+2	+2	0	+3	6
17 Ch	0	0	0	+1	+1	+1	+1	−1	−10	−19	−25	−25	−20	−14	−8	−1	+1	0	0	+1	0	−1	0	0	−5	+4	26
18 Ka	0	−1	−1	0	0	0	+1	+1	−3	−9	−14	−13	−8	−2	+3	+5	+4	+1	−1	−3	−2	−2	−3	−2	−2	+5	19
19 Tu	+1	+3	+3	+4	+4	+4	+3	−4	−12	−14	−10	−2	+3	+5	+5	+3	+2	0	0	0	−1	0	0	0	0	+5	19
20 Lu	0	0	0	0	0	0	0	−1	−6	−9	−8	−4	+2	+9	+13	+12	+9	+4	0	0	0	0	0	0	+1	+5	22
21 He	0	0	+1	0	−1	0	+1	0	−2	−1	+6	+15	+20	+22	+21	+15	+6	0	−2	−2	−1	−1	0	0	+4	+3	24
22 AT	0	0	0	+1	+1	+2	+2	+2	+2	+6	+13	+19	+23	+22	+18	+12	+7	+2	−1	−2	−2	−2	−1	−1	+5	+7	25
23 Ho	0	0	0	+1	+1	+1	+1	+1	−1	−1	+3	+9	+14	+15	+13	+10	+6	+3	0	−1	−1	−2	0	−1	+3	+7	17
24 Te	0	0	0	0	+1	+1	−1	−3	−3	+5	+13	+18	+18	+14	+8	+3	+1	+1	+1	0	−1	−1	−1	0	+3	0	21
25 Al	0	0	0	0	+1	+2	+2	+2	+6	+16	+26	+32	+32	+27	+18	+11	+8	+5	+2	+1	0	−1	−1	−1	+8	+5	33
26 SJ	0	0	0	0	+2	+3	+4	+7	+10	+14	+15	+13	+10	+6	+1	−2	−4	−3	−2	−2	−2	−1	−1	−1	+3	+6	19
27 An	0	0	0	+1	+3	+4	+4	+5	+11	+24	+41	+51	+52	+44	+34	+24	+18	+13	+9	+7	+4	+2	+2	+2	+15	+4	52
28 FP																											
29 Mo	0	+3	+5	+6	+6	+7	+8	+11	+23	+35	+45	+49	+47	+39	+28	+17	+9	+5	+3	+2	+2	+1	+1	0	+15	+5	49
30 Ba	0	+1	+4	+6	+7	+9	+12	+19	+29	+39	+46	+50	+48	+40	+25	+18	+7	+2	+1	0	−1	−1	−1	−1	+15	+4	51
31 El	0	+2	+1	+1	+2	+3	+5	+9	+12	+16	+23	+29	+30	+27	+21	+13	+7	+4	+3	+2	0	−1	−1	−1	+9	+3	31
32 Hu	0	0	0	+2	+3	+5	+6	+9	+35	+60	+85	+93	+86	+73	+53	+35	+20	+7	+2	0	−1	−1	−1	−1	+24	+7	94
33 Ap	0	0	0	+1	+2	+2	+3	+6	+12	+21	+30	+34	+35	+30	+21	+13	+6	+1	0	0	−1	0	0	0	+9	+4	36
34 Tn	0	0	+2	+3	+3	+4	+5	+6	+7	+8	+10	+15	+19	+16	+10	+4	0	−2	−2	−3	−3	−3	−1	0	+4	+2	22
35 Mr	−1	0	+1	+3	+5	+5	+6	+7	+6	+6	+9	+15	+20	+18	+13	+11	+5	+1	0	−1	−2	−2	−2	−1	+5	+5	22
36 LQ	0	0	0	+1	+3	+5	+6	+8	+16	+29	+41	+47	+45	+33	+22	+11	+4	0	−1	−1	−1	−2	−1	−1	+11	+7	49
37 Va	0	−1	+3	+3	+3	+5	+6	+8	+13	+24	+38	+49	+52	+43	+32	+25	+16	+9	+5	+3	+1	+1	0	0	+14	+1	53
38 Wa	0	+1	+2	+3	+3	+4	+5	+5	+5	+4	+3	+1	−3	−5	−5	−4	−2	−1	+1	+1	+1	+1	+1	0	+1	+4	10
39 Pi	−1	0	0	0	+1	+3	+4	+7	+9	+15	+20	+23	+23	+18	+13	+7	+2	−1	−2	−3	−4	−4	−3	−2	+5	+6	27
40 Ca	0	+1	+2	+3	+3	+4	+5	+6	+6	0	−5	−10	−11	−9	−6	−3	−1	0	0	−1	−2	−2	−2	−1	−1	+3	17
41 To	+1	0	0	0	+1	+1	+3	+5	+3	−2	−10	−15	−17	−13	−7	−3	−1	−1	0	+1	+1	+2	+1	0	−2	+6	22
42 Am	+1	+1	0	0	0	+1	+3	+4	+2	−3	−10	−17	−20	−18	−13	−8	−3	−1	0	0	+1	+2	+1	+1	−3	+4	24
43 Mg	0	+1	+1	+2	+3	+5	+8	+8	+5	−5	−15	−20	−21	−19	−20	−15	−9	−6	−7	−5	−4	−5	−5	−2	−5	+9	29
44 OS	+1	+1	+2	+1	0	0	+2	+3	+2	−7	−17	−25	−27	−21	−14	−6	−2	−2	−3	−4	−2	−1	−1	0	−5	+3	30

TABLE 6. THE NORTH COMPONENT OF Sq, D-MONTHS 1932–3

unit = 1γ

L.T. =	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	M	N	R
1 Le	0	−1	−1	0	+1	+3	+5	+5	+2	−1	−5	−8	−8	−5	−2	0	+2	+3	+3	+2	+2	+2	+1	+1	0	+1	13
2 Sl	+1	0	−1	−1	0	+2	+3	+4	+4	+1	−4	−6	−6	−5	−2	0	0	0	+1	+1	+1	+1	+1	+1	0	+2	10
3 Lo	0	0	−1	0	0	+2	+4	+5	+4	0	−4	−7	−7	−4	−1	0	0	+1	+2	+2	+1	+1	+1	+1	0	+1	12
4 Si	−1	0	0	+1	+2	+2	+1	+1	−1	−2	−3	−3	−3	−2	0	+1	+2	+3	+2	+1	0	0	−1	−1	0	0	6
5 RS	0	−1	−1	0	+1	+3	+5	+6	+4	0	−5	−9	−9	−5	−1	0	0	+1	+2	+1	+1	+2	+2	+1	0	+1	15
6 Za	0	0	0	0	−1	−1	0	+1	+2	+2	−1	−2	−2	−3	−1	0	+1	0	0	0	+1	+1	+1	+1	0	0	5
7 Es	0	0	0	+1	+2	+5	+7	+6	+3	−2	−8	−11	−10	−7	−3	−1	+1	+2	+3	+3	+2	+2	+2	+1	0	+1	18
8 Me	0	0	−1	0	+2	+3	+2	+1	0	−4	−8	−7	−6	−2	+2	+3	+4	+4	+4	+3	+2	0	+1	0	0	+2	12
9 Zo	0	0	0	0	+1	+2	+3	+4	+3	0	−4	−6	−5	−1	+2	+4	+3	+1	0	−1	−1	−1	−1	−1	0	+4	10
10 DB	+1	−1	−2	0	+2	+3	+6	+6	+5	−1	−6	−10	−10	−7	−3	−1	0	+2	+3	+3	+3	+3	+3	+2	0	+1	16
11 Ab	0	0	0	+1	+2	+4	+6	+6	+4	−1	−6	−11	−12	−9	−6	−3	−1	0	+1	+1	+1	+1	+1	0	−1	+2	18
12 Ma	0	−2	−1	−1	+1	+2	+4	+5	+5	+3	−2	−5	−6	−5	−3	−3	0	+1	+2	+1	+1	+1	0	0	0	+2	11
13 VJ	0	−1	−1	0	+2	+3	+6	+8	+6	0	−5	−9	−9	−6	−5	−2	0	+1	+2	+3	+2	+2	+1	+1	0	+1	17
14 Ty	0	0	+1	+2	+2	+2	+2	+4	+4	0	−4	−7	−6	−2	−1	+1	+2	+2	−1	−2	−1	−1	0	0	0	−1	11
15 Ag	0	0	+1	+2	+2	+3	+4	+5	+2	−5	−12	−17	−17	−11	−5	+1	+4	+6	+5	+4	+2	+1	0	0	−1	+4	23
16 Eb.	−1	−1	−1	0	+1	+5	+7	+9	+9	+5	−2	−3	−4	−4	−4	−3	0	+1	+2	+1	+2	+2	+1	0	+1	+3	13
17 Ch	0	+1	+2	+4	+5	+6	+6	+6	+4	−2	−9	−15	−17	−12	−6	−1	+5	+6	+6	+4	+2	+2	+1	0	0	+3	23
18 Ka	0	0	0	0	+1	+1	+3	+7	+8	+7	+2	0	−1	0	+2	+4	+6	+4	+2	0	0	−1	−1	−1	+2	+7	9
19 Tu	0	0	+2	+2	+3	+5	+5	+3	+2	−1	−5	−7	−7	−5	−1	+2	+5	+3	+1	0	−1	−2	−3	−2	0	+5	12
20 Lu	0	−1	0	0	0	+1	+4	+7	+11	+10	+7	+6	+5	+5	+5	+6	+4	+2	0	0	0	0	−1	0	+3	+6	12
21 He	+1	+2	0	0	0	0	+4	+8	+12	+17	+17	+14	+13	+12	+7	+2	0	0	+1	+1	0	+1	+1	+1	+5	+2	17
22 AT	0	0	+1	+2	+3	+5	+7	+11	+14	+18	+22	+23	+22	+18	+13	+9	+3	+1	0	−1	−1	−1	−1	0	+7	+6	24
23 Ho	0	+1	+1	+1	+2	+2	+4	+6	+8	+8	+10	+13	+15	+15	+12	+9	+7	+5	+2	+1	0	−1	−1	−1	+5	+6	16
24 Te	−1	0	0	0	+1	+2	+4	+7	+6	+5	+5	+6	+6	+7	+7	+6	+6	+4	+1	0	−1	−1	−2	−1	+3	+1	9
25 Al	0	0	+2	+3	+3	+3	+5	+9	+15	+20	+25	+28	+26	+20	+14	+9	+6	+4	+2	+1	0	0	0	0	+8	+4	28
26 SJ	0	−1	0	+2	+3	+4	+6	+10	+15	+17	+15	+11	+3	−5	−7	−6	−3	0	+2	+2	+1	+1	0	0	+3	+4	24
27 An	0	0	+1	+1	+2	+3	+5	+10	+18	+29	+41	+48	+47	+38	+31	+22	+14	+9	+6	+4	+3	+2	0	0	+14	+3	48
28 FP	+20	+22	+24	+26	+29	+32	+34	+39	+53	+66	+79	+77	+62	+48	+30	+18	+9	+2	0	+2	+5	+9	+14	+17	+30	+8	79
29 Mo	−1	−1	0	0	0	+1	+4	+10	+17	+25	+32	+33	+30	+22	+17	+12	+5	+1	−2	−2	−3	−4	−4	−3	+8	+5	37
30 Ba	0	+2	+4	+5	+7	+9	+12	+18	+27	+37	+45	+47	+41	+33	+24	+13	+5	+2	+1	+1	0	−1	−1	−1	+14	+8	48
31 El	+1	0	0	0	0	+1	+2	+4	+8	+14	+20	+24	+22	+18	+13	+9	+6	+3	+2	0	−1	0	0	0	+6	+4	25
32 Hu	0	0	+1	+2	+3	+4	+7	+16	+28	+48	+65	+74	+70	+50	+36	+22	+13	+7	+3	+2	+1	+1	0	0	+19	+4	74
33 Ap	0	−1	−1	0	+1	+1	0	+4	+12	+23	+37	+43	+40	+32	+25	+15	+7	+1	+1	+1	0	0	−1	−1	+10	+9	44
34 Tn	0	+1	+1	0	0	−1	0	0	+2	+5	+10	+12	+10	+9	+6	0	−4	−4	−4	−4	−4	−4	−3	−2	+1	+3	16
35 Mr	−1	0	0	0	−1	−1	−2	−1	0	+1	+7	+11	+9	+9	+8	+3	−1	−2	−2	−2	−2	−3	−3	−2	+1	+4	14
36 LQ	0	−1	−2	0	0	+2	+5	+9	+16	+29	+38	+43	+40	+32	+23	+15	+10	+5	+3	0	0	0	0	0	+11	+5	45
37 Va	+1	0	0	0	+2	+3	+4	+9	+15	+23	+34	+39	+41	+38	+29	+21	+17	+10	+4	+1	+1	−1	−2	0	+12	+8	43
38 Wa	−1	0	0	0	0	+1	+3	+1	−2	−2	−2	−1	+1	+1	0	−1	−1	0	0	+1	+1	0	0	0	0	+4	5
39 Pi	0	−2	−1	0	+1	+2	+4	+5	+8	+13	+19	+24	+27	+23	+18	+13	+8	+3	0	−1	−1	−1	0	0	+7	+5	29
40 Ca	0	0	0	0	0	0	+2	+3	+1	−4	−8	−11	−10	−8	−7	−5	−4	−3	−3	−3	−3	−3	−2	−1	−3	+4	14
41 To	+1	0	0	−1	0	0	+2	0	−3	−11	−18	−18	−15	−8	−1	+4	+5	+3	+1	+2	+2	+3	+2	+2	−2	+4	23
42 Am	+2	+1	0	0	0	+2	+2	−1	−8	−17	−23	−24	−19	−10	−2	+6	+7	+6	+6	+5	+4	+4	+4	+3	−2	+5	31
43 Mg	0	+1	+2	+3	+5	+7	+7	+2	−7	−15	−18	−20	−18	−17	−12	−9	−9	−6	−4	−3	−3	−2	−1	−1	−5	+6	27
44 OS	+1	−1	0	0	0	−1	−5	−7	−13	−21	−26	−27	−22	−14	−7	−4	−7	−7	−5	−4	−2	−2	−2	+1	−7	+5	28

TABLE 7. THE NORTH COMPONENT OF Sq, J-MONTHS 1933

unit = 1γ

L.T. =	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	*M*	*N*	*R*
1 Le	0	−2	−3	−2	0	−1	−5	−10	−17	−25	−31	−33	−29	−20	−12	−5	+3	+8	+10	+10	+8	+5	+3	+2	−6	+1	43
2 Sl	0	0	−1	−2	−1	−1	−4	−10	−17	−25	−33	−35	−32	−25	−15	−6	−1	+1	+1	+2	+3	+3	+2	+1	−8	+2	38
3 Lo	0	−2	−3	−3	−2	−3	−6	−12	−19	−27	−33	−34	−30	−22	−13	−5	−1	+2	+3	+5	+5	+4	+2	+1	−8	+1	39
4 Si	0	+3	+4	+2	0	−1	−4	−8	−14	−21	−25	−24	−21	−14	−7	0	+5	+6	+4	+1	−1	−1	0	0	−5	0	31
5 RS	+1	−1	−3	−3	−2	−1	−4	−10	−18	−27	−32	−33	−30	−22	−14	−6	−1	+1	+3	+3	+3	+3	+2	+1	−8	+1	36
6 Za	+2	+1	0	0	0	0	−1	−4	−10	−18	−22	−24	−21	−16	−11	−7	−4	0	+1	+1	+1	+3	+3	+3	−5	+1	27
7 Es	−1	−3	−4	−4	−3	−3	−6	−11	−19	−25	−30	−32	−29	−21	−13	−5	0	+4	+5	+5	+4	+2	0	0	−8	+1	37
8 Me	−1	−4	+2	+1	0	0	−2	−6	−13	−21	−26	−28	−22	−17	−11	−4	+2	+11	+9	+2	+2	+1	+1	+1	−5	+6	39
9 Zo	0	−1	−2	−2	−1	+1	0	−6	−17	−26	−31	−31	−24	−15	−8	−3	+1	+1	0	−2	−2	−1	0	0	−7	+2	32
10 DB	+2	0	−1	−2	−1	+1	−2	−8	−16	−23	−26	−27	−25	−20	−14	−6	−1	+2	+3	+4	+5	+4	+4	+3	−6	+2	32
11 Ab	+1	−1	−3	−3	−2	−1	−4	−9	−15	−21	−24	−25	−24	−21	−15	−8	−5	−1	+2	+3	+3	+2	+1	+1	−7	+1	28
12 Ma	+1	−1	−2	−4	−2	−1	−4	−10	−16	−21	−23	−21	−19	−16	−12	−8	−4	−2	0	+2	+3	+3	+2	+2	−6	0	26
13 VJ	0	−2	−4	−5	−4	−2	−5	−9	−16	−21	−22	−21	−17	−15	−13	−9	−6	−3	−1	+1	+2	+1	0	0	−7	+2	24
14 Ty	−1	0	0	−1	+1	+3	+2	−2	−10	−17	−21	−19	−15	−9	−4	−3	−4	−5	−4	−3	−2	−1	0	−1	−5	+2	24
15 Ag	0	−1	−2	−2	−1	+1	+2	−1	−9	−20	−28	−26	−18	−8	−4	+8	+11	+8	+5	+3	+2	0	0	+1	−3	+1	39
16 Eb	+4	+3	+1	0	0	+1	−1	−5	−10	−13	−10	−4	+2	+5	+2	+1	+1	0	+2	+4	+6	+6	+5	+4	0	+2	19
17 Ch	0	−1	−1	−2	−1	+1	+1	−2	−13	−23	−26	−22	−11	−3	+3	+7	+8	+5	+3	+2	+1	+1	0	0	−3	+3	34
18 Ka	0	0	0	0	0	+1	+2	−1	−6	−10	−12	−10	−6	−3	+1	+2	+2	0	−2	−2	−2	−1	0	0	−2	+2	14
19 Tu	−1	+1	+1	+1	+1	+1	0	−6	−12	−12	−6	0	+6	+9	+7	+3	−1	−3	−4	−3	−2	−1	−1	−1	−1	+2	21
20 Lu	0	0	−1	−1	−1	−1	0	−3	−7	−9	−7	−2	+5	+9	+10	+9	+5	0	−3	−3	−1	−1	0	0	0	+4	19
21 He	0	+1	+1	0	0	0	+1	−1	−6	−6	−2	+6	+14	+17	+18	+14	+7	−2	−6	−4	−2	−1	0	0	+2	+3	24
22 AT	0	+1	+1	+1	+2	+2	+2	+3	+5	+9	+16	+21	+23	+20	+15	+10	+2	−3	−5	−3	−2	−1	0	0	+5	+4	28
23 Ho	0	+1	+2	+2	+1	0	+1	+1	+4	+7	+10	+12	+14	+15	+13	+9	+5	+2	0	−1	−2	−2	0	0	+4	+3	17
24 Te	0	0	+1	+1	+1	0	−2	−4	−3	+4	+11	+15	+15	+13	+10	+6	+2	0	+1	+2	+2	+1	0	0	+3	+1	19
25 Al	0	0	0	0	+1	+2	+2	+4	+13	+23	+30	+32	+30	+24	+17	+11	+5	+1	0	0	−1	−1	0	0	+8	+4	33
26 SJ	0	0	0	0	0	+1	+3	+5	+7	+10	+13	+12	+8	+4	0	−2	−4	−4	−2	−1	0	−1	−1	0	+2	+2	17
27 An	0	−2	−1	+2	+2	+2	+4	+12	+20	+30	+39	+46	+46	+41	+33	+21	+11	+5	+2	+1	0	−1	−1	0	+13	+5	48
28 FP	+19	+21	+22	+24	+27	+28	+35	+42	+47	+52	+57	+57	+50	+41	+30	+17	+7	+1	−1	+1	+5	+10	+13	+16	+26	+6	58
29 Mo	0	0	+2	+2	+2	+3	+7	+16	+26	+39	+46	+47	+41	+35	+24	+12	+2	−3	−3	−4	−3	−3	−2	−1	+12	+5	51
30 Ba	0	+1	+3	+5	+7	+9	+13	+21	+31	+39	+48	+50	+44	+33	+22	+10	+1	−2	−1	0	0	0	0	0	+14	+4	52
31 El	0	+1	+1	+2	+2	+4	+7	+13	+17	+19	+22	+23	+22	+18	+14	+8	+2	0	0	0	0	−1	−1	−1	+7	+2	24
32 Hu	0	0	+1	+3	+4	+5	+9	+17	+29	+45	+62	+72	+72	+63	+46	+29	+15	+6	+2	+1	0	−1	−1	0	+20	+3	73
33 Ap	0	+1	+2	+2	+3	+4	+5	+11	+16	+20	+23	+25	+23	+20	+16	+10	+5	+2	+1	0	0	0	0	0	+8	+3	25
34 Tn	0	+1	+3	+4	+5	+6	+7	+12	+15	+16	+16	+16	+15	+13	+8	+2	−2	−3	−2	−2	−3	−2	−2	−1	+5	+2	19
35 Mr	0	+1	+3	+5	+6	+8	+11	+16	+19	+16	+14	+14	+15	+16	+13	+8	+3	0	+1	0	0	−1	−1	−1	+7	+3	20
36 LQ	0	0	+1	+2	+3	+6	+7	+15	+21	+25	+29	+34	+32	+24	+12	+5	+2	+1	0	+1	0	−1	−2	−1	+9	+4	36
37 Va	0	−1	−1	−1	0	+1	+3	+7	+12	+20	+29	+34	+34	+29	+21	+13	+11	+4	+2	+2	+1	+2	+1	0	+9	+6	35
38 Wa	−1	−1	0	+1	+2	+3	+4	+6	+8	+12	+13	+11	+8	+4	−1	−6	−7	−5	−1	+1	+1	0	−1	−1	+2	+1	20
39 Pi	0	+1	+1	+2	+3	+5	+7	+10	+15	+18	+20	+18	+15	+11	+7	+5	+4	+3	+1	+1	+1	−1	−2	−1	+6	+3	22
40 Ca	−1	0	0	+1	+1	+2	+4	+7	+12	+11	+6	0	−4	−8	−10	−7	−3	+1	+1	−1	−2	−3	−3	−2	0	+2	22
41 To	0	+1	+1	+2	+3	+4	+5	+7	+12	+13	+10	+4	−3	−9	−11	−11	−6	−1	+1	+1	0	0	0	0	+1	+2	24
42 Am	0	0	0	0	0	+2	+3	+4	+8	+10	+9	+3	−4	−10	−12	−9	−3	0	0	0	0	0	0	0	0	+3	22
43 Mg	0	0	0	+1	+2	+3	+4	+6	+4	+2	−3	−7	−9	−6	+1	+3	+2	+1	−1	−1	−2	−2	−2	−1	0	+1	15
44 OS	0	0	0	0	0	+1	+3	+4	+4	+3	−1	−5	−5	−3	+1	+4	+3	+1	0	−2	−2	−2	−2	−1	0	+2	9

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Table 8. The east component of Sq, E-months 1932–3

unit = 1γ

L.T. =	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	M	N	R
1 Le	0	0	0	+2	+3	+4	+5	+7	+9	+8	+2	−6	−15	−21	−19	−14	−9	−8	−8	−6	−5	−3	0	0	−3	−2	30
2 Sl	+1	0	−1	+1	+3	+4	+5	+7	+9	+7	+3	−7	−16	−22	−22	−17	−11	−6	−5	−5	−3	−1	0	+1	−3	−2	31
3 Lo	0	0	+1	+2	+3	+4	+5	+7	+9	+8	+2	−8	−16	−23	−23	−17	−9	−6	−5	−5	−4	−2	0	+1	−3	−2	32
4 Si	+2	+3	+4	+6	+7	+8	+10	+15	+18	+13	+4	−6	−13	−18	−17	−15	−11	−7	−4	−1	0	0	0	0	0	−1	36
5 RS	0	0	+1	+2	+3	+4	+5	+8	+11	+10	+2	−8	−18	−25	−24	−16	−8	−5	−6	−5	−3	−2	+1	+1	−3	−2	36
6 Za	0	0	−1	0	−1	+2	+3	+6	+8	+9	+5	−4	−8	−18	−23	−22	−17	−9	−6	−4	−3	−2	−1	0	−4	−2	32
7 Es	−1	0	0	+1	+1	+2	+4	+8	+11	+10	+3	−8	−19	−25	−23	−17	−11	−9	−8	−6	−5	−3	−1	−2	−4	−2	36
8 Me	0	0	+2	+5	+7	+9	+10	+15	+16	+14	+6	−5	−14	−17	−16	−12	−8	−6	−4	−2	0	+2	+1	+1	0	−1	33
9 Zo	0	+1	+2	+3	+4	+5	+6	+9	+13	+14	+9	+1	−10	−18	−21	−17	−10	−5	−2	−3	−3	−2	−2	−1	−1	0	35
10 DB	0	0	0	+1	+3	+2	+4	+9	+14	+12	+2	−12	−24	−29	−27	−17	−10	−7	−7	−6	−3	−2	−1	0	−4	0	43
11 Ab	0	0	0	0	+1	+2	+4	+7	+12	+12	+3	−9	−19	−27	−26	−18	−11	−7	−6	−6	−4	−2	0	−1	−4	−2	39
12 Ma	−1	0	0	+1	+2	+2	+4	+7	+12	+12	+5	−7	−22	−28	−26	−17	−11	−6	−7	−5	−4	−3	−1	−1	−4	−2	40
13 VJ	0	0	−1	0	+2	+2	+4	+8	+13	+14	+5	−9	−22	−29	−27	−19	−11	−7	−7	−6	−3	−3	−1	0	−4	−1	43
14 Ty	0	+1	+3	+4	+3	+3	+2	+6	+8	+9	+7	−2	−12	−20	−21	−17	−11	−6	−2	−1	−2	−2	−2	−1	−2	+3	30
15 Ag	0	0	0	+2	+4	+6	+9	+15	+18	+15	+6	−6	−17	−21	−20	−14	−9	−5	−2	−1	−1	0	0	0	−1	−1	39
16 Eb	0	+1	−1	0	+2	+2	+4	+9	+13	+13	+2	−13	−27	−32	−30	−20	−14	−7	−8	−6	−4	−4	−3	−2	−5	−1	45
17 Ch	0	0	+1	+2	+4	+6	+10	+17	+21	+19	+8	−5	−15	−20	−19	−15	−9	−4	−2	−1	−1	−1	0	0	0	−2	41
18 Ka	0	+1	+2	+3	+4	+4	+4	+9	+13	+17	+12	0	−12	−21	−21	−14	−6	0	+2	0	0	0	0	0	0	+5	38
19 Tu	0	0	+1	+2	+4	+5	+9	+18	+22	+17	+2	−12	−18	−19	−16	−11	−5	−1	0	0	+1	+1	0	0	0	+2	41
20 Lu	0	0	+1	+2	+2	+3	+5	+10	+18	+20	+10	−1	−14	−21	−19	−12	−4	+1	+1	0	−1	−1	−1	−1	0	−1	41
21 He	+1	+1	0	0	0	+1	+4	+10	+17	+15	+3	−10	−21	−26	−23	−14	−7	−5	−6	−6	−5	−3	−1	0	−3	−2	43
22 AT	−1	0	+1	+2	+1	+1	+5	+13	+19	+19	+9	−4	−16	−20	−17	−10	−2	+2	0	−1	−1	−2	−2	−2	0	+1	39
23 Ho	0	0	+2	+4	+5	+7	+10	+20	+27	+26	+15	+1	−9	−13	−12	−7	−3	−1	0	+1	+1	0	0	0	+3	0	40
24 Te	0	−1	0	+2	+4	+6	+12	+22	+23	+10	−4	−12	−14	−13	−9	−3	−1	−1	0	0	+1	0	0	0	+1	0	37
25 Al	0	+1	+2	+2	+1	0	+3	+11	+14	+9	0	−11	−20	−20	−11	−5	+1	+2	0	−2	−2	−2	−1	−1	−1	−1	34
26 SJ	0	0	+1	+2	+2	+4	+8	+12	+15	+14	+9	+3	−2	−7	−11	−10	−8	−6	−3	−1	0	0	0	0	+1	−2	26
27 An	0	0	0	+1	+1	+1	+4	+9	+12	+7	+1	−7	−15	−17	−13	−6	0	+3	+2	+1	0	−1	−2	−1	−1	0	29
28 FP																											
29 Mo	0	0	0	0	0	+2	+3	0	−11	−17	−16	−12	−2	+7	+11	+13	+9	+2	−2	−3	−3	−2	0	0	−1	−2	30
30 Ba	0	−3	−4	−3	−6	−8	−10	−14	−24	−31	−26	−14	+2	+16	+26	+29	+28	+24	+20	+16	+12	+7	+4	+2	+2	+2	60
31 El	0	0	0	−1	−1	−1	−2	−7	−23	−28	−23	−15	−6	+2	+8	+11	+9	+3	−3	−3	−1	0	0	0	−3	−1	39
32 Hu	0	0	0	+1	+2	+4	+3	+1	+1	+5	+10	+15	+18	+20	+19	+14	+6	+1	0	+1	0	+1	+1	0	+5	0	20
33 Ap	0	0	0	+1	+2	+4	+3	0	−7	−12	−13	−9	−1	+6	+10	+11	+8	+5	+5	+5	+4	+2	+1	+1	+1	+1	24
34 Tn	0	0	0	0	0	0	+3	+3	−5	−14	−15	−11	−2	+8	+16	+19	+16	+8	+1	−1	0	0	0	0	+1	0	34
35 Mr	0	−1	−1	−2	−2	−2	−1	−1	−9	−15	−17	−12	−1	+11	+19	+20	+16	+8	+3	+1	+1	+1	+2	+2	+1	−1	37
36 LQ	0	0	0	0	+1	+3	+6	+1	−5	−7	−1	+9	+17	+25	+22	+14	+6	+2	−2	0	+1	+1	+1	+1	+4	0	32
37 Va	+1	0	−1	0	0	+2	+6	−1	−13	−19	−17	−9	+2	+11	+8	+5	0	−3	−3	−1	+1	+1	+2	+2	−1	0	30
38 Wa	0	0	0	0	0	0	−1	−6	−15	−20	−22	−18	−5	+8	+17	+19	+15	+10	+5	+4	+3	+2	0	0	0	−1	41
39 Pi	0	0	0	0	+1	+4	+7	0	−13	−23	−16	0	+18	+31	+34	+24	+15	+6	+1	+2	+2	+2	+2	+1	+4	0	57
40 Ca	0	0	−1	−1	−1	−2	−3	−8	−16	−25	−28	−20	−7	+7	+15	+16	+11	+5	+1	+1	+2	+2	+2	+1	−2	0	44
41 To	0	0	0	0	0	−1	−2	−8	−16	−22	−19	−8	+7	+21	+28	+25	+17	+10	+6	+3	+2	+1	0	0	+2	+2	50
42 Am	−1	−1	0	0	0	0	−2	−6	−14	−21	−21	−15	−2	+10	+19	+19	+14	+9	+5	+3	+2	0	−1	−2	0	0	40
43 Mg	+1	0	0	0	0	−1	0	−7	−20	−27	−22	−9	+6	+20	+29	+27	+20	+13	+8	+8	+7	+6	+5	+2	+3	+6	56
44 OS	+1	0	−1	−1	−2	−3	−2	−3	−8	−13	−12	−6	+5	+16	+20	+20	+14	+8	+4	+3	+3	+2	+1	+1	+2	0	33

TABLE 9. THE EAST COMPONENT OF Sq, D-MONTHS 1932–3

unit = 1γ

L.T. =	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	*M*	*N*	*R*
1 Le	+3	+1	0	−1	0	0	+1	+2	+2	+3	+1	−2	−6	−8	−6	−4	−2	−1	−1	0	+2	+5	+6	+5	0	−2	14
2 Sl	+3	+1	+1	0	0	0	+1	+2	+3	+3	+2	−1	−5	−7	−7	−5	−1	−2	−1	0	+1	+2	+3	+4	0	−1	11
3 Lo	+3	+2	0	−1	−1	0	0	+1	+3	+3	+2	−1	−6	−8	−6	−4	−2	−2	−1	0	+1	+3	+5	+5	0	−1	13
4 Si	0	−1	−2	−2	+1	+2	+2	+4	+6	+5	+2	−1	−4	−7	−7	−5	−2	0	+1	+1	+1	+1	+1	+1	0	0	13
5 RS	+3	+2	0	−1	−1	0	+1	+2	+4	+4	+3	−2	−6	−9	−8	−4	−1	−1	0	+1	+1	+4	+5	+5	0	−1	14
6 Za	+2	+2	+1	0	−1	−1	−1	−1	+2	+3	+4	+2	−3	−4	−5	−4	−1	0	0	−1	0	+1	+1	+2	0	+1	9
7 Es	+2	0	−1	−1	−1	0	+1	+2	+4	+5	+3	−2	−7	−10	−7	−4	−2	−1	0	+1	+3	+5	+5	+5	0	−2	15
8 Me	0	−1	−3	−2	0	+2	+4	+5	+7	+8	+4	−3	−7	−8	−7	−5	−3	−1	+1	+2	+3	+2	+1	0	0	+1	16
9 Zo	+1	+2	+2	0	−1	−2	−2	−1	+2	+6	+6	+2	−3	−6	−5	−2	0	+1	+1	+1	0	0	+1	+1	0	0	12
10 DB	+3	+1	−2	−2	−1	0	0	+2	+5	+6	+2	−4	−10	−11	−6	−3	−2	0	0	+1	+3	+5	+6	+5	0	+3	17
11 Ab	+3	+2	0	0	−1	0	+1	+2	+4	+6	+4	−2	−8	−11	−9	−5	−2	0	0	+1	+2	+4	+4	+3	0	−3	17
12 Ma	+3	+1	0	−1	−1	0	0	+1	+3	+5	+2	−4	−9	−12	−10	−6	−2	−1	0	0	+1	+3	+4	+3	−1	−2	17
13 VJ	+3	+2	−2	−1	−1	0	+1	+1	+4	+8	+5	−2	−10	−11	−10	−4	−1	0	+1	+2	+3	+4	+5	+4	0	−2	19
14 Ty	0	0	0	0	0	0	−1	−2	0	+4	+4	0	−5	−8	−7	−5	−1	0	0	−1	−1	−1	−1	0	−1	+2	12
15 Ag	0	−2	−3	−2	−1	+1	+3	+5	+10	+11	+7	0	−7	−12	−11	−7	−4	−1	0	+2	+3	+3	+3	+2	0	−1	23
16 Eb	+2	0	0	−1	−1	−1	0	+1	+4	+7	0	−7	−13	−16	−13	−7	−4	−3	−1	0	+1	+1	+2	+2	−2	−2	23
17 Ch	+1	−2	−4	−3	−2	0	+2	+5	+11	+13	+11	+4	−6	−11	−12	−9	−5	−2	0	+1	+2	+3	+2	+1	0	0	25
18 Ka	0	0	0	0	0	0	−1	−2	+3	+10	+10	+4	−4	−11	−10	−6	0	+3	+3	+1	0	0	0	0	0	+1	21
19 Tu	−1	−2	−2	−2	−1	0	0	+5	+11	+14	+12	+3	−7	−11	−11	−6	−1	+2	+5	+5	+4	+3	+2	+1	+1	−1	25
20 Lu	0	0	0	0	−1	−2	−3	−4	−1	+10	+10	+4	−3	−8	−8	−5	0	+3	+2	+1	0	0	0	0	0	+2	18
21 He	+1	0	−1	−3	−5	−5	−4	−4	+2	+7	+7	−4	−10	−10	−8	−6	−4	−3	−2	0	+1	+1	+2	+2	−2	−2	17
22 AT	0	0	−1	−1	−2	−3	−6	−6	−2	+6	+9	+5	0	−2	−4	−3	0	+3	+2	+1	0	0	0	0	0	−4	15
23 Ho	0	−1	−1	0	+1	+1	+1	+2	+8	+13	+13	+7	−1	−5	−7	−4	−1	+3	+5	+5	+4	+2	0	0	+2	+2	20
24 Te	0	0	0	0	+1	+2	+2	+3	+7	+13	+13	+10	+3	−4	−6	−4	0	+3	+5	+6	+6	+6	+4	+2	+3	+2	19
25 Al	0	0	−1	−3	−5	−6	−7	−7	−4	+2	+2	0	−3	−1	+1	0	+1	+2	+1	+1	+1	+1	+1	0	−1	−1	9
26 SJ	+2	+1	0	0	−1	0	0	0	+6	+16	+22	+16	+6	−2	−7	−8	−5	0	+2	+3	+4	+5	+4	+3	+3	−1	30
27 An	+1	0	0	0	0	−1	−4	−6	−4	+1	+7	+6	+2	0	0	+1	+4	+6	+4	+3	+2	+1	+1	+1	+1	0	13
28 FP	0	0	−2	−3	−3	−4	−7	−15	−14	−4	+7	+10	+12	+14	+16	+15	+14	+15	+14	+12	+7	+3	+1	0	+4	−1	31
29 Mo	+1	0	−2	−3	−4	−7	−11	−18	−16	−6	+2	+8	+12	+14	+14	+7	+2	0	0	+1	+2	+3	+2	+1	0	−2	32
30 Ba	0	−2	−4	−8	−12	−16	−25	−33	−34	−27	−15	0	+13	+26	+34	+37	+34	+29	+23	+18	+14	+9	+6	+3	+3	−1	71
31 El	+1	0	−1	−2	−4	−7	−12	−19	−20	−13	−5	+2	+8	+11	+9	+8	+5	+2	0	+2	+3	+4	+3	+2	−1	−2	31
32 Hu	−2	−3	−4	−5	−5	−5	−9	−11	−2	+10	+21	+29	+30	+24	+18	+12	+5	+2	+2	+3	+4	+3	+2	0	+5	+1	41
33 Ap	0	−1	−1	0	0	−1	−7	−17	−23	−22	−10	+6	+18	+24	+24	+18	+12	+7	+6	+7	+6	+4	+2	+1	+2	+3	47
34 Tn	0	0	0	−1	−2	−3	−6	−11	−12	−7	−1	+7	+16	+21	+21	+16	+10	+6	+4	+2	+2	+2	+2	+1	+3	−1	33
35 Mr	0	−2	−3	−5	−6	−7	−10	−19	−17	−12	−6	−2	+10	+13	+15	+13	+9	+5	+4	+5	+3	+3	+3	+2	0	−3	34
36 LQ	−1	−3	−5	−5	−5	−5	−10	−11	−5	+5	+17	+27	+29	+26	+24	+17	+8	+4	+2	+2	+3	+3	+2	+1	+5	0	40
37 Va	+2	−1	−3	−3	−4	−4	−4	−12	−14	−7	+1	+9	+15	+13	+4	−3	−5	−4	0	+3	+4	+4	+4	+3	0	0	29
38 Wa	+2	+1	0	−1	−3	−7	−15	−24	−29	−29	−22	−7	+6	+17	+23	+23	+18	+12	+9	+6	+6	+5	+4	+2	0	0	52
39 Pi	−1	−4	−6	−7	−6	−8	−15	−19	−15	−5	+13	+28	+36	+38	+33	+20	+9	+6	+4	+4	+5	+3	+1	+1	+5	0	57
40 Ca	+2	+1	−1	−2	−4	−7	−13	−20	−26	−23	−15	−5	+7	+15	+18	+16	+12	+8	+6	+6	+6	+6	+5	+4	0	−1	44
41 To	0	−1	−1	−1	−2	−6	−14	−25	−32	−33	−24	−4	+17	+33	+32	+33	+27	+17	+10	+6	+4	+2	+1	+1	+2	+1	66
42 Am	0	−1	−1	−2	−3	−6	−13	−22	−29	−30	−20	0	+14	+25	+32	+30	+25	+18	+12	+8	+6	+5	+3	+1	+2	+2	62
43 Mg	−2	−4	−7	−7	−9	−13	−20	−21	−21	−19	−5	+9	+22	+23	+18	+14	+12	+10	+8	+4	+2	+1	+1	0	0	−2	44
44 OS	0	−3	−6	−10	−14	−16	−20	−21	−19	−14	−7	+1	+13	+21	+23	+18	+13	+8	+7	+7	+7	+6	+3	+2	0	0	44

TABLE 10. THE EAST COMPONENT OF Sq, J-MONTHS 1933

unit = 1γ

L.T. =	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	M	N	R
1 Le	0	+2	+5	+7	+11	+17	+21	+24	+24	+19	+9	−4	−14	−20	−20	−17	−13	−11	−10	−9	−9	−7	−3	−1	0	0	44
2 Sl	+1	+2	+3	+6	+12	+17	+22	+24	+22	+16	+6	−7	−19	−27	−29	−24	−15	−8	−5	−5	−5	−5	−4	−2	−1	0	53
3 Lo	+1	+1	+3	+7	+11	+16	+20	+23	+22	+17	+9	−5	−18	−26	−28	−24	−14	−9	−6	−5	−6	−6	−5	−2	−1	0	51
4 Si	0	+1	+1	+4	+9	+16	+23	+27	+28	+23	+12	−2	−16	−26	−29	−27	−21	−14	−8	−3	−1	0	0	−1	0	+6	57
5 RS	0	+1	+3	+6	+6	+14	+21	+23	+23	+19	+9	−6	−20	−28	−27	−21	−14	−8	−5	−5	−6	−6	−2	0	−1	0	51
6 Za	+1	+2	+4	+5	+7	+12	+18	+21	+22	+19	+11	0	−13	−21	−24	−23	−18	−9	−4	−3	−3	−2	−1	0	0	0	46
7 Es	0	+2	+4	+6	+10	+17	+22	+24	+25	+21	+8	−6	−17	−22	−23	−19	−14	−10	−9	−7	−6	−4	−2	−1	0	0	48
8 Me	0	0	+1	+2	+5	+15	+22	+27	+28	+23	+9	−8	−19	−24	−26	−26	−19	−9	−5	−3	−1	−1	0	0	0	+3	54
9 Zo	0	+2	+4	+5	+9	+15	+22	+28	+28	+24	+12	−3	−18	−27	−30	−26	−19	−10	−4	−1	−1	−3	−3	−2	0	+1	58
10 DB	0	+1	+2	+3	+7	+14	+20	+23	+24	+18	+5	−11	−26	−31	−27	−21	−14	−8	−6	−6	−7	−5	−3	−1	−2	+1	55
11 Ab	+1	+3	+4	+5	+8	+15	+21	+25	+26	+21	+10	−5	−18	−24	−24	−20	−13	−8	−6	−5	−5	−4	−2	0	0	0	50
12 Ma	0	+1	+3	+5	+7	+13	+21	+24	+26	+21	+10	−8	−23	−30	−27	−22	−15	−8	−5	−5	−6	−5	−3	−1	−1	0	56
13 VJ	0	0	+2	+2	+5	+12	+20	+24	+27	+21	+8	−8	−22	−28	−26	−21	−13	−7	−5	−5	−5	−4	−2	0	−1	−2	55
14 Ty	0	+1	+2	+4	+6	+11	+18	+22	+21	+16	+6	−7	−19	−25	−26	−23	−16	−8	−1	0	−1	−2	−2	−1	−1	+4	48
15 Ag	0	0	0	+1	+5	+12	+19	+26	+25	+18	+4	−12	−24	−29	−28	−23	−15	−8	−3	−1	−2	−1	−1	0	−2	−1	55
16 Eb	0	+2	+3	+4	+6	+11	+19	+25	+26	+20	+7	−10	−25	−29	−29	−22	−13	−6	−4	−3	−3	−3	−2	0	−1	0	55
17 Ch	0	+1	+2	+2	+5	+12	+20	+28	+30	+23	+8	−9	−22	−28	−27	−21	−14	−7	−2	−1	−1	−1	0	0	0	+1	58
18 Ka	0	+1	+3	+5	+6	+8	+13	+24	+29	+24	+11	−4	−17	−25	−27	−24	−16	−8	−1	+1	−1	−1	−1	−1	0	0	56
19 Tu	0	0	+1	+3	+5	+9	+19	+28	+32	+25	+8	−10	−22	−26	−24	−18	−11	−5	−3	−3	−3	−2	−1	0	0	0	58
20 Lu	−1	0	+2	+2	+3	+7	+18	+30	+33	+26	+9	−9	−20	−26	−25	−19	−11	−4	−1	−2	−3	−3	−3	−2	0	−1	59
21 He	0	+2	+2	+2	+3	+6	+19	+27	+30	+21	+5	−12	−22	−28	−26	−20	−11	−5	−5	−5	−4	−3	−1	−1	−1	0	58
22 AT	0	+1	+3	+4	+3	+7	+20	+30	+32	+21	+4	−9	−19	−21	−20	−15	−5	0	0	−2	−3	−3	−3	−2	+1	−2	53
23 Ho	−2	−1	0	+3	+5	+7	+14	+29	+31	+22	+8	−5	−15	−18	−16	−11	−6	−3	−3	−2	−2	−3	−3	−2	+1	−1	49
24 Te	0	0	+1	+2	+4	+7	+16	+28	+28	+18	+2	−11	−18	−19	−16	−12	−7	−5	−6	−5	−4	−3	−2	−1	0	+1	47
25 Al	+2	+3	+4	+5	+5	+7	+16	+25	+25	+15	0	−15	−25	−26	−20	−12	−2	+2	+1	−3	−4	−3	−1	0	0	0	51
26 SJ	0	+1	+1	+2	+3	+6	+15	+25	+25	+15	+4	−4	−12	−17	−19	−16	−10	−6	−5	−4	−4	−2	−1	0	0	−1	44
27 An	−1	0	+1	+3	+5	+7	+16	+25	+26	+16	+3	−8	−16	−18	−16	−12	−7	−2	−3	−3	−4	−4	−3	−2	0	+1	44
28 FP	+2	+3	+3	+4	+5	+7	+13	+17	+11	0	−9	−15	−19	−19	−16	−11	−8	−6	−5	−5	−3	−1	0	+1	−2	−2	36
29 Mo	−1	0	+1	+1	+1	+3	+6	+11	0	−11	−16	−16	−8	+1	+6	+6	+4	0	−6	−8	−7	−5	−3	−2	−2	−1	27
30 Ba	0	0	0	0	0	0	+3	+5	−2	−11	−15	−15	−10	−2	+5	+12	+17	+15	+9	+5	+3	+1	+1	0	+1	−3	32
31 El	0	0	0	+1	+2	+2	+5	+7	−1	−11	−19	−20	−15	−8	0	+6	+5	−1	−6	−7	−4	−3	−2	−1	−3	0	27
32 Hu	0	0	+2	+3	+5	+7	+11	+15	+15	+12	+10	+9	+10	+12	+12	+9	+3	−3	−4	−3	−2	−2	−1	−1	+5	0	19
33 Ap	−3	−3	−2	+1	+4	+7	+9	+11	+11	+5	−3	−8	−10	−10	−5	+3	+6	+4	0	−3	−3	−4	−4	−4	0	−4	21
34 Tn	0	+1	+1	+2	+2	+3	+5	+10	+5	−6	−13	−13	−9	−2	+5	+11	+10	+4	−3	−5	−5	−4	−2	−1	0	0	24
35 Mr	0	0	+1	+1	+1	+1	+2	+6	+2	−6	−10	−10	−7	−1	+6	+12	+12	+4	−2	−4	−4	−2	−1	0	0	0	22
36 LQ	0	0	+1	+2	+4	+6	+9	+14	+10	+3	−2	−1	+7	+11	+12	+9	+2	−2	−3	−3	−1	0	0	0	+3	−1	17
37 Va	0	+1	+1	+2	+3	+5	+8	+13	+4	−2	−7	−5	0	+1	+2	0	−4	−6	−5	−3	−3	−2	−1	0	0	−1	20
38 Wa	−3	−2	0	0	0	0	+1	+2	0	−7	−11	−12	−9	−4	+2	+7	+6	+2	−1	−3	−4	−4	−4	−4	−2	−1	19
39 Pi	−1	0	0	+1	+3	+5	+8	+11	+9	−3	−11	−7	+7	+16	+16	+10	+2	−4	−4	−3	−2	−1	−1	−1	+2	0	27
40 Ca	0	0	0	0	+1	+1	+1	+2	0	−9	−16	−16	−9	0	+6	+10	+7	0	−4	−3	−2	−2	−1	0	−1	0	26
41 To	−3	−2	−1	0	+1	+1	+1	+2	+2	−1	−7	−9	−8	−1	+6	+12	+12	+5	+1	−1	−2	−3	−3	−3	0	0	21
42 Am	−4	−3	−2	0	+1	+1	+1	0	0	−3	−8	−10	−6	0	+6	+9	+7	+4	+1	−1	−3	−4	−5	−5	−1	0	19
43 Mg	0	0	+1	+2	+3	+3	+4	+5	+2	−3	−5	+1	+7	+10	+8	+4	+2	+1	+1	0	0	+1	+1	0	+2	0	15
44 OS	−1	−1	0	0	0	+2	+2	+2	+1	−1	−3	0	+4	+7	+7	+5	+1	0	0	0	0	0	0	−1	+1	0	10

TABLE 11. THE VERTICAL COMPONENT OF Sq, E-MONTHS 1932–3

unit = 1γ

L.T. =	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	*M*	*N*	*R*
1 Le	−9	−7	−4	−1	+1	+1	+1	+1	0	−1	−2	−3	−4	−4	−2	+2	+3	+3	+2	+2	+1	0	−1	−5	−1	+3	12
2 Sl	−3	−3	−1	0	0	+1	+1	0	−3	−6	−7	−6	−2	+1	+3	+2	+2	+1	+1	+1	0	0	−2	−3	−1	+1	10
3 Lo	−3	−4	−4	−2	−1	0	+1	+1	+1	−1	−4	−7	−7	−5	−1	+2	+3	+3	+2	+1	+1	+1	0	−1	−1	+2	10
4 Si	0	−1	−1	−1	−3	−5	−3	−3	−3	−4	−4	−4	−3	0	+2	+4	+5	+4	+4	+4	+4	+2	+2	+2	0	+1	10
5 RS	−1	−2	−2	−1	0	+1	+2	+2	+1	−3	−9	−12	−11	−7	−2	+2	+4	+3	+2	+2	+3	+2	+1	0	−1	+1	16
6 Za																											
7 Es	−1	−1	0	0	+1	+1	+1	+1	0	−2	−4	−6	−8	−7	−2	+2	+4	+4	+3	+2	+2	+3	+2	+1	0	0	12
8 Me	−3	−7	−9	−4	−2	−2	−3	−1	−1	−1	−1	−2	−3	−2	−1	+1	+3	+5	+9	+7	+5	+4	+5	+2	0	−2	18
9 Zo																											
10 DB	0	−1	−1	0	0	0	0	+2	+3	0	−3	−6	−8	−7	−3	+1	+3	+4	+3	+2	+3	+3	+2	+2	0	−1	12
11 Ab	+1	0	0	0	+1	+2	+2	+3	+2	−2	−6	−10	−11	−9	−4	+2	+4	+5	+4	+4	+3	+3	+3	+2	0	−1	16
12 Ma	+1	0	0	0	0	0	0	0	−1	−4	−7	−10	−12	−10	−5	0	+3	+3	+2	+2	+2	+2	+2	+2	−1	0	15
13 VJ	0	0	0	+1	+1	+1	+1	+1	+1	−3	−7	−11	−14	−12	−8	−3	0	+1	0	0	+1	+1	+1	+1	−2	−1	15
14 Ty	0	0	0	0	0	0	0	0	0	0	−2	−4	−5	−4	−3	−2	−1	−1	−1	−1	−1	−1	0	0	−1	−1	5
15 Ag	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	0	+1	+1	+1	+1	+1	+1	+1	0	0	0	0	−2	2
16 Eb	0	0	0	0	0	0	0	+1	0	−4	−12	−19	−20	−18	−12	−8	−3	−2	−1	0	0	0	0	0	−4	0	21
17 Ch	−1	−1	0	0	0	0	+1	+1	−1	−3	−7	−10	−8	−5	−2	0	+2	+3	+2	+1	+1	+1	0	0	−1	−1	13
18 Ka	0	0	0	+1	+1	+2	+2	+1	−2	−11	−17	−16	−11	−5	−2	−1	−2	−3	−4	−4	−2	−1	−1	−1	−3	−3	19
19 Tu	0	0	0	0	0	0	+1	+2	+1	−5	−11	−13	−12	−10	−6	−2	0	+1	+1	+1	0	0	0	0	−2	−1	15
20 Lu	0	0	0	+1	+1	+1	+2	+3	+3	−2	−8	−12	−14	−13	−9	−5	−2	−1	0	0	0	0	0	0	−2	0	17
21 He	0	0	−1	0	0	0	0	+1	−2	−8	−15	−17	−15	−11	−4	+1	+2	0	−1	−1	−1	0	0	0	−3	−1	19
22 AT	0	0	+1	+1	0	0	+1	+1	−1	−7	−14	−18	−17	−14	−10	−7	−4	−3	−3	−2	−1	0	0	0	−4	0	19
23 Ho	−1	−1	0	0	0	0	+3	+7	+8	+2	−8	−16	−19	−17	−13	−9	−6	−5	−5	−4	−3	−3	−2	−2	−4	−2	27
24 Te	−1	−1	0	0	0	+1	+2	+2	−7	−10	−12	−12	−11	−10	−8	−5	−6	−6	−4	−3	−2	−2	−2	−1	−4	−1	14
25 Al	0	0	0	0	0	+1	+3	+5	−2	−13	−19	−23	−20	−13	−4	+1	+1	−3	−3	−3	−3	−1	−1	0	−4	−5	28
26 SJ	0	0	0	0	0	0	0	−3	−6	−8	−9	−9	−7	−6	−5	−4	−2	−1	0	0	+1	+1	+1	0	−2	+1	10
27 An	0	0	0	−1	−1	−1	−2	−3	−7	−14	−20	−23	−22	−14	−7	0	+3	+3	+2	+2	+1	+2	+2	+1	−4	0	26
28 FP																											
29 Mo	−1	−3	−4	−5	−6	−7	−8	−9	0	+9	+16	+16	+11	+1	−5	−7	−4	0	+2	+2	+2	+1	+1	0	0	+1	25
30 Ba	0	+1	+1	+2	+2	+2	+2	+1	0	+5	+14	+21	+24	+23	+16	+9	+4	0	−1	−1	−1	−1	−1	−1	+5	+4	25
31 El	0	+1	0	0	0	0	−1	−1	0	+5	+10	+12	+11	+12	+12	+8	+3	+1	+1	+2	+1	+1	+1	0	+3	0	13
32 Hu	0	0	+1	+1	+2	+2	+2	+1	+1	−1	−2	−2	−3	−2	−3	−6	−7	−6	−3	−2	−1	−1	0	0	−1	+1	9
33 Ap																											
34 Tn	0	−2	−4	−6	−7	−9	−10	−10	−10	−7	−3	+2	+6	+8	+8	+8	+7	+6	+5	+5	+5	+4	+2	+1	0	−2	18
35 Mr	0	0	+1	+1	0	0	0	0	−3	−5	−4	+1	+7	+13	+15	+13	+8	+4	+2	0	0	−1	−1	−1	+2	+1	20
36 LQ	0	0	0	+1	+1	+2	+2	−3	−4	+6	+17	+23	+25	+23	+15	+7	+1	−2	−1	0	+1	+1	+1	+1	+5	−1	29
37 Va	0	−1	0	0	0	0	−1	−4	−1	+5	+9	+11	+9	+4	−1	−3	−3	−2	−1	0	0	0	0	0	+1	−1	15
38 Wa	0	+1	+2	+3	+3	+2	0	−5	−8	−8	0	+11	+22	+28	+26	+18	+9	+2	−1	−2	−3	−3	−3	−2	+4	+1	36
39 Pi	0	0	0	0	+1	+1	+1	0	0	+7	+14	+19	+23	+20	+13	+8	+4	+2	+2	+2	+1	0	0	−1	+5	0	24
40 Ca	+1	+4	+6	+6	+5	+8	+11	+6	−3	−4	+3	+10	+20	+23	+19	+13	+4	−2	−4	−4	−3	−2	−1	0	+5	+3	27
41 To	0	0	0	0	0	−1	0	0	−1	−1	+1	+5	+8	+7	+5	+2	0	−1	0	+1	0	0	0	0	+1	+2	9
42 Am	0	0	+1	+1	+1	+1	0	0	−1	−2	−1	+1	+2	+3	+3	+2	0	0	−2	−2	−2	−2	−1	−1	0	+1	5
43 Mg	0	0	0	+1	+1	+4	+3	0	−5	−9	−2	+2	+11	+15	+12	+7	+5	0	−2	0	0	0	0	0	+2	+1	24
44 OS																											

TABLE 12. THE VERTICAL COMPONENT OF Sq, D-MONTHS 1932–3

unit = 1γ

L.T. =	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	M	N	R
1 Le	+1	0	0	0	0	0	0	0	+1	+2	+3	+3	+3	+3	+4	+4	+3	+3	+3	+3	+3	+4	+3	+2	+2	+2	4
2 Sl	0	0	0	0	0	0	0	0	0	−1	0	+1	+2	+2	+2	+2	+2	+2	+2	+2	+2	0	−1	−1	+1	+1	3
3 Lo	0	0	0	0	0	0	0	0	0	0	0	0	0	+2	+3	+3	+3	+3	+3	+2	+2	+2	+1	0	+1	0	3
4 Si	−1	−3	−4	−2	−3	−3	−2	−1	−1	−1	−1	0	+1	+1	+2	+3	+3	+2	+2	+2	+2	+1	+1	0	0	+5	7
5 RS	−1	−2	−2	−1	0	0	0	0	0	−2	−4	−4	−3	0	+3	+3	+2	+2	+1	+2	+2	+1	+1	0	0	0	7
6 Za																											
7 Es	+1	0	0	0	0	0	0	0	0	0	0	0	0	+1	+2	+3	+3	+2	+2	+2	+2	+3	+2	+1	+1	0	3
8 Me	−5	−6	−7	−7	−5	−4	−2	−1	0	0	+1	+1	+2	+3	+4	+4	+3	+4	+4	+4	+3	+2	+1	0	0	−7	11
9 Zo																											
10 DB	+1	0	0	−1	−1	−1	−1	−1	0	+1	0	0	0	+1	+3	+4	+3	+3	+2	+2	+2	+2	+3	+2	+1	0	5
11 Ab	+1	0	0	−1	0	0	+1	0	+1	0	−1	−3	−2	0	+3	+3	+3	+3	+3	+3	+2	+2	+2	+2	+1	−1	6
12 Ma	0	0	0	0	0	−1	−1	−1	−2	−4	−5	−6	−4	−2	0	+1	+1	0	+1	0	0	0	0	0	−1	0	7
13 VJ	0	0	0	0	0	+1	0	−1	−1	−1	−3	−4	−4	−2	+1	+2	+2	+2	+2	+1	+1	+1	+1	+1	0	−1	6
14 Ty	0	0	0	0	0	0	0	0	+1	0	0	−1	−1	0	+1	+2	+1	0	−1	−1	−1	−1	0	0	0	−1	3
15 Ag	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	0	+1	+1	+1	0	0	0	0	0	0	0	0	−1	2
16 Eb	0	0	0	+1	−1	−2	−2	−4	−4	−8	−15	−17	−15	−13	−7	−3	−1	−2	0	0	0	−1	−1	0	−4	+3	18
17 Ch	0	+1	+1	0	0	0	0	0	0	−2	−5	−7	−5	−2	+1	+3	+3	+2	+1	+1	+1	0	0	0	0	−1	10
18 Ka	0	0	0	+1	+1	+1	+2	+3	+1	−5	−11	−13	−10	−5	−2	0	0	−3	−4	−3	−2	−1	−1	−1	−2	+1	16
19 Tu	0	0	0	0	0	0	0	+1	0	−1	−3	−7	−9	−8	−5	−2	0	+2	+2	+2	+1	+1	0	0	−1	−2	11
20 Lu	0	0	0	+1	+1	0	0	+1	0	−2	−5	−8	−11	−9	−7	−4	−2	−1	−1	−1	−1	−1	0	0	−2	+1	12
21 He	0	0	0	−1	0	−1	−1	−1	−2	−3	−8	−12	−10	−6	−3	−1	0	0	0	0	0	0	0	0	−2	0	12
22 AT	−1	0	0	0	0	+1	+1	+1	+2	0	−7	−11	−13	−13	−10	−6	−3	−2	−2	−2	−2	−1	−1	−1	−3	0	15
23 Ho	0	0	0	0	0	0	0	0	+3	+3	0	−7	−13	−13	−10	−8	−4	0	0	0	0	0	0	0	−2	0	16
24 Te	−1	0	0	+1	+1	+1	+1	+2	+3	+2	−1	−5	−8	−8	−7	−4	−2	0	0	0	0	0	−1	−1	−1	−1	11
25 Al	+1	+1	0	0	0	−2	−2	−1	+2	+2	−2	−6	−4	−1	−1	−1	+1	+1	+1	+1	+1	+1	+1	+1	0	−3	7
26 SJ	0	0	0	0	0	0	0	−1	−2	−3	−6	−9	−10	−8	−6	−3	−1	−1	−1	−1	0	0	0	0	−2	−1	10
27 An	+1	0	+1	0	0	0	0	0	0	−1	−4	−9	−12	−10	−7	−2	+1	+3	+3	+2	+2	+2	+2	+1	−1	0	15
28 FP	0	+1	+3	+4	+6	+7	+8	+12	+19	+26	+31	+28	+21	+14	+7	0	−6	−9	−9	−7	−6	−4	−3	−2	+6	+1	40
29 Mo	0	+2	−3	−5	−5	−5	−5	−2	+3	+7	+7	+5	+2	+2	−3	−3	−1	+1	+2	+2	+2	+1	+1	0	0	0	12
30 Ba	0	0	0	+1	+1	+1	0	−2	+1	+7	+16	+22	+23	+20	+16	+11	+5	0	−2	−1	−1	−1	−1	0	+5	+4	25
31 El	0	0	0	0	0	0	−1	+1	+4	+8	+13	+15	+14	+13	+12	+7	+4	+2	+2	+2	+1	+1	0	0	+4	0	16
32 Hu	0	0	+1	+2	+2	+2	+2	+3	+7	+9	+10	+12	+9	+7	+5	+3	0	−2	−1	0	+1	+1	0	0	+3	+1	14
33 Ap																											
34 Tn	−1	−2	−3	−4	−6	−7	−8	−10	−10	−6	−4	−2	+1	+3	+6	+7	+8	+9	+9	+8	+7	+5	+3	+1	0	0	19
35 Mr	0	0	0	0	−1	−1	−2	−4	−3	−1	−1	+4	+5	+7	+6	+4	+3	+2	+1	+1	+1	+1	+1	0	+1	0	11
36 LQ	0	0	0	0	+1	+1	−1	0	+5	+11	+18	+21	+19	+15	+7	0	−3	−2	0	+2	+1	0	0	0	+4	−1	24
37 Va	0	0	0	+1	+1	0	0	0	+2	+4	+5	+4	+2	−2	−5	−5	−3	−3	−2	−1	0	0	0	0	0	0	10
38 Wa	0	0	0	0	−1	−3	−8	−11	−8	+1	+15	+26	+32	+30	+25	+15	+6	+1	−1	−1	0	0	−1	0	+5	+2	43
39 Pi	0	0	0	0	0	0	0	+2	+6	+12	+16	+15	+15	+15	+8	+4	+2	+2	+1	+1	+1	+1	0	0	+4	+1	16
40 Ca	+3	+4	+5	+4	+2	+5	+5	0	−2	+6	+14	+19	+22	+19	+14	+8	+2	−2	−3	−3	−1	−1	−1	+1	+5	+1	25
41 To	0	0	0	−1	−1	−1	−2	−1	0	+2	+4	+4	+5	+6	+6	+4	+1	0	−1	0	0	+1	+1	+1	+1	+2	8
42 Am	0	0	0	0	0	0	0	−1	−2	0	+2	+4	+5	+6	+5	+4	+2	0	−1	0	0	0	0	0	+1	0	8
43 Mg	−1	−1	0	0	0	0	+1	0	0	+2	+10	+17	+20	+17	+9	+8	+8	+6	+5	+4	+2	+1	+1	+1	+5	+2	21
44 OS																											

TABLE 13. THE VERTICAL COMPONENT OF Sq, J-MONTHS 1933

unit = 1γ

L.T. =	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	M	N	R
1 Le	−2	−3	−1	0	+2	+4	+4	+3	+2	0	−3	−6	−9	−8	−5	−2	+1	+4	+5	+5	+4	+3	+2	0	0	+1	14
2 Sl	−1	0	+1	+2	+2	+2	+1	−1	−5	−8	−9	−7	−3	0	+2	+3	+4	+3	+3	+3	+3	+2	0	−1	0	−1	13
3 Lo	−1	−2	−1	0	+2	+3	+2	+1	−1	−5	−8	−10	−10	−7	−3	0	+2	+4	+4	+4	+3	+2	+1	0	−1	+1	14
4 Si	+1	+1	−1	−1	0	0	0	0	−4	−9	−11	−11	−7	−3	+1	+6	+9	+11	+10	+8	+6	+6	+6	+4	+1	+8	22
5 RS	0	−1	0	+1	+3	+4	+3	+1	−2	−6	−11	−15	−15	−10	−3	+1	+4	+5	+6	+4	+3	+2	+2	+1	−1	0	21
6 Za																											
7 Es	−1	−1	0	+1	+2	+3	+3	+2	0	−3	−8	−12	−13	−10	−5	−1	+2	+5	+5	+4	+2	+1	+1	0	−1	0	18
8 Me	−6	−15	−9	−6	−5	+1	+3	+3	+1	−1	−3	−6	−6	−2	+1	+5	+8	+9	+10	+7	+5	+3	+2	−1	0	−5	25
9 Zo																											
10 DB	0	−1	−1	0	+1	+2	+3	+3	+3	+1	−3	−8	−11	−10	−5	−1	+3	+5	+6	+5	+4	+3	+2	+1	0	−2	17
11 Ab	0	0	0	+1	+3	+4	+2	+1	0	−4	−9	−14	−16	−13	−6	0	+4	+6	+6	+4	+2	+1	+1	0	−1	0	22
12 Ma	0	0	0	0	+1	+2	+1	+1	−1	−4	−8	−12	−14	−11	−6	0	+3	+5	+5	+4	+2	+2	+2	+1	−1	0	19
13 VJ	0	−1	0	0	+2	+3	+3	+2	+1	−2	−7	−12	−16	−15	−10	−4	0	+2	+3	+2	+1	+1	+1	0	−2	−1	19
14 Ty	0	0	0	+1	+1	+1	+1	0	−2	−4	−7	−9	−9	−7	−5	−3	−2	−1	0	−1	−1	−1	−1	−1	−2	−1	10
15 Ag	0	0	0	0	0	+1	+1	0	0	−1	−1	−2	−2	−2	−1	0	0	+1	+1	+1	+1	+1	+1	0	0	+1	3
16 Eb	0	0	0	0	0	+2	+1	−1	−4	−9	−13	−18	−18	−18	−15	−8	−2	+2	+3	+3	+1	0	0	−1	−4	−2	21
17 Ch	0	0	0	−1	0	+2	+3	+2	0	−4	−9	−13	−12	−9	−5	+1	+4	+5	+4	+2	+2	+1	0	0	−1	−1	18
18 Ka	0	0	0	+1	+1	+3	+3	−3	−11	−17	−18	−16	−11	−7	−5	−3	−2	−3	−3	−3	−1	0	0	0	−4	−1	21
19 Tu	0	0	0	0	0	+1	+3	+4	+1	−6	−12	−13	−12	−10	−7	−4	0	+2	+2	+1	0	0	0	0	−2	−1	17
20 Lu	0	0	0	0	0	+1	+2	+1	−2	−8	−13	−16	−16	−14	−11	−7	−4	−2	−1	−1	−1	0	0	0	−4	−1	18
21 He	0	0	0	0	+1	+2	+3	+1	−4	−12	−20	−22	−20	−14	−9	−2	+2	+2	−2	−2	−1	0	0	0	−4	0	25
22 AT	0	0	0	0	0	+1	+3	+1	−4	−11	−15	−16	−14	−12	−9	−7	−3	−3	−2	−2	−1	0	0	0	−4	0	19
23 Ho	−1	0	0	+1	+2	+3	+7	+12	+8	−1	−9	−14	−15	−12	−7	−4	−2	−2	−3	−3	−3	−2	−1	−1	−2	−1	27
24 Te	0	0	+1	+1	+1	+2	+3	+3	−1	−5	−7	−8	−7	−5	−5	−4	−3	−4	−3	−2	−2	−1	−1	−1	−2	0	11
25 Al	0	0	−1	−1	−1	+2	+6	+5	−5	−19	−27	−27	−22	−14	−7	0	+3	+1	−3	−3	−3	−2	−1	0	−5	0	33
26 SJ	0	0	0	0	0	+1	+1	−2	−7	−9	−8	−4	−3	−2	0	+2	+3	+2	+1	0	0	0	0	0	−1	−1	12
27 An	0	0	0	−1	−2	−1	−1	−3	−10	−17	−21	−22	−20	−15	−10	−5	−1	+2	+2	+2	+2	+2	+1	+1	−5	+1	24
28 FP	−1	0	0	+1	+1	+2	+2	+1	−1	0	+5	+7	+8	+7	+3	−1	−4	−6	−7	−6	−5	−4	−3	−2	0	+1	15
29 Mo	0	−2	−3	−3	−4	−5	−4	−6	−2	+6	+13	+13	+9	+3	−5	−8	−7	−3	0	+2	+1	+1	+2	+1	0	−1	21
30 Ba	−1	0	0	0	0	0	+2	+3	+2	+5	+9	+12	+12	+9	+5	+3	0	−5	−7	−6	−5	−4	−3	−2	+1	−2	19
31 El	0	0	0	0	0	0	0	−1	−4	−3	+3	+8	+11	+13	+12	+8	+4	0	−1	0	0	0	0	0	+2	0	17
32 Hu	0	0	0	+1	+1	+2	+3	+3	+1	−3	−6	−7	−7	−6	−7	−8	−8	−6	−3	−1	0	0	0	0	−2	0	11
33 Ap																											
34 Tn	0	−1	−2	−3	−4	−5	−6	−6	−7	−5	−2	+1	+3	+5	+5	+6	+5	+4	+3	+3	+3	+2	+2	+1	0	+2	13
35 Mr	0	0	0	+1	+1	+1	+1	0	−1	−3	−3	0	+2	+7	+9	+10	+8	+2	−1	−2	−2	−1	−1	0	+1	+1	13
36 LQ	0	0	0	0	0	+1	+2	+3	+1	+2	+6	+15	+21	+19	+13	+5	−2	−4	−2	−1	0	0	0	0	+3	0	25
37 Va	0	0	0	0	0	0	0	−1	−2	+2	+5	+8	+8	+5	+4	+2	0	−1	−1	−1	−1	−1	0	0	+1	−2	10
38 Wa	0	+1	+2	+2	+2	+2	+2	+3	+1	−3	−3	−1	+3	+8	+10	+8	+3	−1	−3	−3	−3	−3	−2	−1	+1	−1	13
39 Pi	0	0	0	+1	+1	+1	+1	+2	+2	+3	+6	+11	+14	+11	+6	+2	0	−1	0	0	0	0	0	0	+2	0	15
40 Ca	+5	+8	+9	+7	+5	+8	+14	+14	+8	+3	+1	+3	+7	+8	+10	+9	+3	0	0	−1	−1	0	0	+3	+5	+1	15
41 To	0	0	0	+1	+1	+1	+2	+3	+5	+5	+4	+4	+5	+4	+2	0	−2	−2	−2	−1	−1	0	0	0	+1	+1	7
42 Am	0	0	+1	+1	+2	+2	+2	+3	+3	+3	+3	+3	+2	+2	+1	0	0	0	−1	−1	0	0	0	0	+1	+1	4
43 Mg	0	0	0	+1	+3	+3	+4	+4	+3	+2	+2	+4	+5	+4	+2	+1	+1	+1	+1	+1	+1	0	0	0	+2	+2	5
44 OS																											

suggested that larger errors were present. Occasionally arithmetical and copying mistakes were detected in the published data.

The main sources of inaccuracy in the reduced data are therefore likely to be (i) observational errors in measurement and temperature correction, (ii) errors arising from interpolation at times of loss of record, (iii) computational errors in rounding off and use of approximate formulae, e.g. in reducing to local time, (iv) undetected arithmetical and copying mistakes, (v) incomplete elimination of disturbance and (vi) incorrect choice of datum-line. Since, however, each reduced value represents as a rule the mean of 20 observations, we can probably assume that our mean values are seldom in error by more than 1 γ or 2γ. On the other hand, for reasons given in the following sections, they do not necessarily represent exactly the typical or 'normal' variation of the Sq field at each observatory.

5. The definition of the Sq field

5·1. *Introduction*

The total magnetic field at the earth's surface is separable in theory into several distinct parts including (i) the main field arising from sources within the earth and subject only to the secular variation, (ii) the disturbance field D, and (iii) the periodic variation fields such as Sq and L. Since the lower atmosphere is practically non-conducting, and the observed air-earth current so small that its magnetic field is negligible, it follows that each of these partial fields is derivable from a potential function defined on and near the earth's surface. The separation of the actually observed surface field into parts corresponding to the above fields is not, however, completely determinate unless these fields are more closely defined. The first stage in separating out the Sq field from the others is to eliminate the D and L fields as far as possible by averaging over a number of quiet days, and then applying a linear correction to remove any non-cyclic change. This is most conveniently done by using the hourly values of the field expressed as deviations from the mean of the days; but it does not necessarily follow that the daily variations actually produced by ionospheric sources, or other causes, are such that the three components of the field all have zero mean values, though previous analyses of the field have generally been based on this assumption. Further, the elimination of D and L in this way is not completely satisfactory, since the field on quiet days shows both irregular and systematic changes from day to day and throughout the year; the effects of these changes on the seasonal mean daily variations are discussed below.

The problem of determining the Sq field in the sense defined above is essentially that of interpolating the field at any epoch over the earth's surface. For this a necessary condition is that the values shall all be measured from a datum line which corresponds to the same basic situation at every station, i.e. a datum line which corresponds to the resultant field arising from the field sources having the same fixed value (e.g. zero) at every station. The fact that the daily mean value at any station often shows considerable changes from one day to the next is an indication that this is not a satisfactory datum line to adopt. This is illustrated by figure 3 which shows the variation of the horizontal component at Huancayo on the international quiet days 5, 6, 7 March 1933. This figure also shows the considerable day-to-day variability of Sq sometimes observed, and the serious effect this could have on the estimate of the non-cyclic change if this were made using daytime hourly values.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

In the present investigation it was found that the most satisfactory fit between the observed variations and a smoothly distributed Sq field was obtained when the variations are expressed as deviations from a suitably chosen night value. The considerations that led to this choice are discussed in § 5·5.

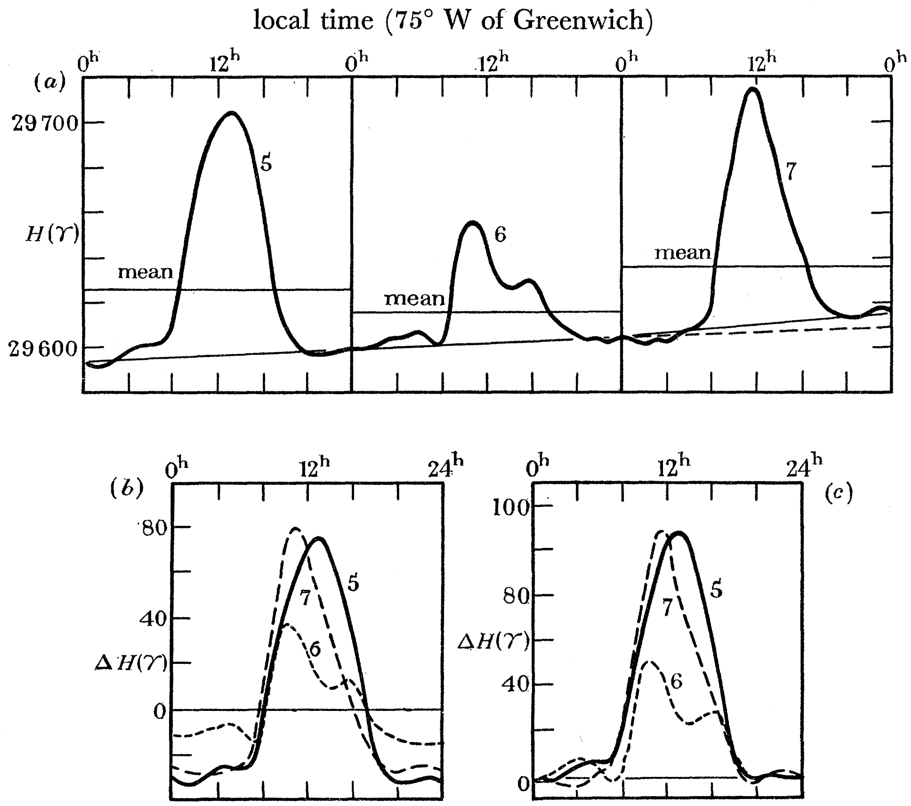


FIGURE 3. The variation of H at Huancayo on 5, 6, 7 March 1933. (*a*) Observed field showing day-to-day variability and non-cyclic change. (*b*) Deviations from daily mean, corrected for non-cyclic change. (*c*) Deviations from night value, corrected for non-cyclic change.

5·2. *The day-to-day variability of Sq*

It is well known that, although the form and intensity of the Sq variations are mainly determined by the geographic positions of the observatory and the time of year, there are often considerable differences in them from one day to the next (Chapman & Bartels 1940, chapters 7 and 19; see also figure 3). This day-to-day variability of Sq affects the problem of defining the 'normal' Sq variation at an observatory. The variability is sometimes a world-wide phenomenon affecting all observatories, and sometimes a regional one, affecting a group of observatories; occasionally it is a purely local one, affecting possibly only one observatory. When interpolating the Sq field over the earth it is particularly important to eliminate the local effects, and this must be done by averaging over a sufficient number of days. Further, the world-wide and regional aspects of the variability make it very desirable that at all observatories the days chosen for this averaging shall be the same. It is usual to take the arithmetic mean of the hourly values for the 5 international quiet days to represent the daily variation for each month, and to average again over the 4 months to obtain the daily variation for a season. Though this method serves to eliminate or at least reduce variability and disturbance effects, it does not necessarily lead to an exact representation of the typical variation occurring on any day. For example, the maximum deviation of a field component may occur at different times on different days, and this would result in the mean daily variation having a maximum deviation that is smaller than that of the typical daily variation.

5·3. *The month-to-month changes of Sq*

The usual practice of dividing the year into one equinoctial and two solstitial 'seasons', each covering 4 months, has been followed in the present investigation. This is the most convenient way of taking account of the main changes in the overall pattern of Sq throughout the year, but it must be noted that there are often considerable differences between individual months of the same season, so that the seasonal mean daily variation does not necessarily represent a really typical variation at any particular time of year. These differences between months of the same season are most readily seen in the mean daily ranges of the various magnetic elements at certain observatories. Thus at San Juan the range of horizontal component was 7γ in September 1932, 15γ in October and 25γ in April 1933. This rather extreme example is associated with the fact that at San Juan there occurs a change of type of the horizontal variation from the northern type with a midday minimum, which was present on 3 September, to the equatorial type with a midday maximum, which was present on 17 September. This emphasizes the care that must be taken in interpreting mean-variation curves. Examination of the monthly mean variations at other observatories, including Huancayo and Tatuoca, as well as the earlier work of Rooney (1945) on those at Honolulu, reveal that at some stations changes in amplitude and form of the Sq variations take place quite rapidly at certain times of the year, *but that these times are not always the same for different observatories*. In view of this last fact and of the nature of the available data, it was considered that no advantage would be obtained in the present analysis by adopting any grouping of the months different from the usual seasonal grouping. It is suggested, however, that when more data are available, a study of these month-to-month changes (which may not be the same in all years) may reveal some interesting facts about the continuous change of the Sq field throughout the year.

5·4. *The removal of non-cyclic change*

The corrections applied to the mean hourly values to remove the non-cyclic change, which is the difference in the values at two epochs differing by 24^{h}, are based on the assumption that the change takes place linearly throughout the day. The principal factors causing the non-cyclic change on any one day are (i) a gradual change due to the smooth recovery of the field from the effects of magnetic storms, (ii) short-period disturbances affecting the field at the two epochs chosen for measuring the differences and (iii) changes of amplitude or form of the daily variation from one day to the next. The last two factors give rise to an irregular part of the non-cyclic change, and unless this part can be eliminated by taking the mean values for a sufficient number of quiet days, the removal of the total non-cyclic change by treating it as a linear change throughout the day may introduce inaccuracies in *all* the hourly values.

The non-cyclic change is usually taken to be the difference of the values occurring at two epochs coinciding with, or close to, successive midnights of either universal or local time. When, as in most cases, the observatory data are in the form of hourly means centred at the half-hours, it is best to use the difference of the means over the two-hour intervals centred at successive midnights as this helps to reduce the effects of disturbance. Some observatories, however, estimate non-cyclic change from the differences of the means over one-hour intervals immediately following successive midnights. We have found that in certain cases these two methods of estimating the non-cyclic change can give significantly different results, and we give an example of this in the next paragraph.

The difference between the universal day and the local day for the particular observatory affects the problem of estimating the non-cyclic change. In studying disturbances, it is more appropriate to use universal time, i.e. Greenwich time, rather than local time, and in fact the international quiet and disturbed days are defined as Greenwich days. In studying the daily variations it has to be remembered that these are in the main local-time variations, though the Sq field also varies with universal time. When the hourly values are averaged for a sufficiently large number of quiet (Greenwich) days and the non-cyclic change is

TABLE 14. THE NON-CYCLIC CHANGE AT WATHEROO DURING THE POLAR YEAR

	horizontal component (γ)			declination (E)			vertical component (γ)		
	A	B	$\frac{1}{2}(A+B)$	A ′	B ′	$\frac{1}{2}(A+B)$ ′	A	B	$\frac{1}{2}(A+B)$
E-months, U.T.	+3·7	+3·8	+3·7	−0·11	−0·16	−0·13	+0·8	+1·7	+1·3
E-months, L.T.	+2·6	+3·7	+3·2	−0·19	−0·04	−0·12	+1·8	+0·2	+1·0
D-months, U.T.	+3·1	+4·5	+3·8	−0·10	+0·25	+0·08	+0·8	+2·4	+1·6
D-months, L.T.	0·0	−0·3	−0·2	−0·17	−0·37	−0·27	+2·3	+2·6	+2·5
J-months, U.T.	+1·5	+1·2	+1·3	−0·10	−0·04	−0·07	−0·6	−0·5	−0·5
J-months, L.T.	+3·2	+2·8	+3·0	−0·19	−0·27	−0·23	+1·8	+1·3	+1·6
Year, U.T.	+2·7	+3·2	+2·9	−0·10	+0·02	−0·04	+0·3	+1·2	+0·8
Year, L.T.	+1·9	+2·1	+2·0	−0·18	−0·23	−0·21	+2·0	+1·4	+1·7

A = Algebraic excess for the hour 23^h to 24^h on the mean quiet day as compared with the corresponding hour on the previous day.

B = Algebraic excess for the hour 0^h to 1^h on the succeeding day as compared with the corresponding hour on the mean quiet day.

removed in the usual way, it is possible to change to the local day simply by a shift of time origin, and the result is not seriously different from that obtained when local days are used from the start. But the number of days which must be averaged to ensure that this is so depends on the longitude of the observatory, the number being usually considerably larger for observatories which lie more than about 90° from the Greenwich meridian. This is because the epochs used for calculating the non-cyclic change then fall within the daylight hours and therefore the effects of the day-to-day variability of Sq become important. This is illustrated by the non-cyclic changes calculated for Watheroo during the Polar Year, using Greenwich days or local days. The results are shown in table 14. This table also shows the effect of estimating the non-cyclic change from (i) the difference in the mean values for the hour 23 to 24 and the mean values for the same hour on the succeeding day, (ii) the difference in the mean values for the hours 0 to 1 of the 2 days, and (iii) the difference in the mean values for the two-hour periods 23 to 1 for the 2 days.

At Watheroo the Greenwich midnight occurs at about 8^h local time, when the daily variation field is large. The table shows the serious differences in the two estimates of non-cyclic change derived from local days and Greenwich days, especially for the J- and D-months. The differences are somewhat reduced by using means over two hours instead of over one hour at the two epochs, but it would evidently be necessary to average over a much larger number of days to remove satisfactorily the effects of disturbance and variability. The estimates of non-cyclic change derived from local night values are probably the most reliable index of the smooth after-storm recovery effect, but unfortunately the observatory data were not usually available in a form suitable for making the estimate in this way.

An examination of the values of the estimated non-cyclic change for each observatory and season shows it to be irregularly distributed over the earth, the values at relatively near observatories often differing appreciably. This is a further indication that the irregular part of the non-cyclic change has not been satisfactorily eliminated at all stations. It follows that there are errors in the reduced hourly values due to wrongly estimated corrections for non-cyclic change. These errors are mainly in the north component and may on occasion exceed 3γ, though in most cases they are definitely less than this.

5·5. *The datum line for measurement of Sq*

5·51. *Theoretical considerations*

Let the potential of the Sq field at any epoch (universal time t) be denoted by $\Omega\,(\theta, \lambda, t)$, where θ, λ are the geographic colatitude and east longitude, and t is measured at the rate of 2π radians per day. Then the north and east components of the field are respectively

$$X = \frac{1}{a}\frac{\partial}{\partial\theta}\Omega(\theta, \lambda, t), \quad Y = -\frac{1}{a\sin\theta}\frac{\partial}{\partial\lambda}\Omega(\theta, \lambda, t). \tag{5·1}$$

Let t' denote the local time at (θ, λ), so that

$$t' = t+\lambda. \tag{5·2}$$

Then

$$\Omega(\theta, \lambda, t) = \Omega(\theta, \lambda, t'-\lambda) = \Omega'(\theta, \lambda, t'), \quad \text{say}, \tag{5·3}$$

and therefore

$$\frac{\partial\Omega}{\partial\theta} = \frac{\partial\Omega'}{\partial\theta}, \quad \frac{\partial\Omega}{\partial\lambda} = \frac{\partial\Omega'}{\partial\lambda}+\frac{\partial\Omega'}{\partial t'}\frac{\partial t'}{\partial\lambda} = \frac{\partial\Omega'}{\partial\lambda}+\frac{\partial\Omega'}{\partial t'}. \tag{5·4}$$

The daily mean values of X and Y can then be written

$$\overline{X} = \frac{1}{2\pi a}\int_0^{2\pi}\frac{\partial\Omega'}{\partial\theta}\,dt' = \frac{1}{2\pi a}\int_0^{2\pi}\frac{\partial\Omega}{\partial\theta}\,dt, \tag{5·5}$$

$$\overline{Y} = -\frac{1}{2\pi a\sin\theta}\int_0^{2\pi}\frac{\partial\Omega'}{\partial\lambda}\,dt'. \tag{5·6}$$

If the field were a strictly local-time field Ω' would be independent of λ, so that

$$\frac{\partial\Omega'}{\partial\lambda} = 0 \quad \text{and} \quad \frac{\partial\Omega'}{\partial\theta} \quad \text{is a function of } \theta \text{ and } t' \text{ only.} \tag{5·7}$$

Hence the mean value of Y, but not that of X, would necessarily be zero in this case. Thus the datum line for Y, but not for X, would be determined. Any change in the datum line for X would simply add a constant zonal field, $X_0(\theta)$ say. This was pointed out by Chapman & Bartels (1940, chapter 7).

If we assume that the daily mean value of X is zero for a strictly local-time field then

$$X = \frac{1}{a}\frac{\partial}{\partial\theta}\Omega'(\theta, t') = X(\theta, t+\lambda). \tag{5·8}$$

Hence

$$\int_0^{2\pi} X\,\mathrm{d}\lambda = \int_0^{2\pi} X\,\mathrm{d}t' = 0, \quad \text{by hypothesis.} \tag{5·9}$$

Hasegawa (1936) found, however, that at a given epoch of universal time, the mean values of X round circles of latitude are not zero, and concluded that the field is not purely a local time one. The non-vanishing of $\int_0^{2\pi} X\,\mathrm{d}\lambda$ indicates a non-zero net flow of the representative current system round circles of latitude.

If we write

$$\Omega'(\theta,\lambda,t') = \Omega'_0(\theta,\lambda) + \Omega'_p(\theta,\lambda,t'), \tag{5·10}$$

$$X'(\theta,\lambda,t') = X'_0(\theta,\lambda) + X'_p(\theta,\lambda,t'), \tag{5·11}$$

$$Y'(\theta,\lambda,t') = Y'_0(\theta,\lambda) + Y'_p(\theta,\lambda,t'), \tag{5·12}$$

where Ω'_p, X'_p and Y'_p are periodic functions of t' with zero mean values, then the quantities which are directly obtainable from the observatory data are $X'_p(\theta, \lambda, t')$ and $Y'_p(\theta, \lambda, t')$ from which Ω'_p can be obtained but not Ω'_0.

If, on the other hand, we write

$$\Omega(\theta,\lambda,t) = \Omega_m(\theta,t) + \Omega_q(\theta,\lambda,t), \tag{5·13}$$

where Ω_q is periodic in λ with mean value zero, then

$$X = \frac{1}{a}\frac{\partial\Omega_m}{\partial\theta} + \frac{\partial\Omega_q}{\partial\theta}, \tag{5·14}$$

$$Y = -\frac{1}{a\sin\theta}\frac{\partial\Omega_q}{\partial\lambda}. \tag{5·15}$$

Hence the mean value of Y with respect to λ is zero. It follows that, though the mean value of Y with respect to t', i.e. $Y'_0(\theta,\lambda)$, is not in general zero, it must be such that the mean value of

$$Y'_0(\theta,\lambda) + Y'_p(\theta,\lambda,t+\lambda)$$

with respect to λ is zero. Since Y'_p is known, this fixes the mean value of Y'_0 with respect to λ, but not $Y'_0(\theta,\lambda)$ itself. There is no corresponding restriction on $X'_0(\theta,\lambda)$ but, since X and Y at any fixed epoch t are derived from the potential Ω, the general relation

$$\frac{\partial X'}{\partial\lambda} + \frac{\partial X'}{\partial t'} + \frac{\partial}{\partial\theta}(Y'\sin\theta) = 0 \tag{5·16}$$

must hold for the separate parts of X' and Y' in (5·11) and (5·12).

It is clear from the above that no further indication as to the correct choice of datum line can be obtained from the measurements at a single observatory. It is necessary to consider the results of interpolating data from all observatories, together with other considerations, to reach a decision about the datum line. Once it has been fixed the values of $X'(\theta,\lambda,t')$ and $Y'(\theta,\lambda,t')$ will be known completely. A knowledge of either X' or Y' will determine $\Omega'_p(\theta,\lambda,t')$, but to determine $\Omega'_0(\theta,\lambda)$ a knowledge of X is clearly required.

5·52. *The measurement of Sq from its night value*

In the early stages of the present investigation the Sq field components were measured at each observatory from the mean of the day, but difficulties in the interpolation soon became evident. These were considerably reduced when the field was measured from a night value. When the east component was measured from the daily mean, it was often found that at a given epoch of universal time the mean values round the latitude circles were not zero, as they should be. These non-zero values were greater than could be reasonably attributed to errors in interpolation and were considerably reduced when the datum line was changed. Again, when attempting to find a potential function to fit both the north and east components, discrepancies were found which could be attributed, in the night hemisphere particularly, to a wrong choice of datum line. These were again largely removed by changing to a night value.

The use of the night value as the datum line is also suggested by other considerations, which apply to middle and low latitudes but not to the auroral zones.

(i) The ionization in the ionosphere is known to decay rapidly after sunset. Hence the ionospheric current systems at night may be expected to be very much smaller than those in daylight hours. This is not found, however, when the field is measured from the mean of the day: the total intensity of the night system is then found to be of the order of one-third, or one-half, of that of the day.

(ii) The changes of the field components during the night are much smaller than during the day, and in some cases, depending on the component and the season, are approximately zero. This confirms the previous deduction about the night current systems.

(iii) The mean value of the day is more affected by the day-to-day variability of Sq and by disturbance than is the night value. The latter may show a steadier change from day-to-day. This is strikingly illustrated in figure 3 by the variation of the horizontal component at Huancayo on the international quiet days 5, 6, 7 March 1933.

In figure 3 (*a*) are plotted the hourly values on these 3 days. The four night values show a practically linear increase from day to day, but the three means do not, owing to the much reduced amplitude on the middle day. In figures 3 (*b*) and (*c*) are plotted the daily variations, corrected for non-cyclic change, as deviations from the daily mean values and from the night values, respectively. In the latter case, the curves show many similarities and differ appreciably only in the amplitude of the midday maximum.

Three possible choices suggest themselves for defining the night value of any particular component.

(i) The value at a certain hour, e.g. midnight. This is unreliable owing to possible disturbance effects.

(ii) The value, if any, at which the component is approximately constant for several hours. This would seem to be the best since this is likely to be due to a zero rather than to a constant current system.

(iii) When there is no such constant value during the night, the mean over several hours, say 21^h to 3^h or 20^h to 4^h, is suitable. A longer interval is not suitable since there are often noticeable post-sunset and pre-dawn changes in the field. Slight modifications may be needed to take account of any obvious disturbance. In some cases the night value is uncertain but it is possible to choose it so that the variation curve is in the best agreement, for the rest of the day, with that of another observatory where the night value is more certain. In other cases an appropriate datum line may involve small non-zero night values, e.g. it is possible that there are some north–south currents in the night hemisphere.

The differences between the night value and the daily mean value are most noticeable in the north component, particularly in the equatorial regions. However, on the assumption that the ionospheric current systems during the night are small, the same criterion must be applied to all three components.

The existence of a potential function gives a test of this, or any other, assumption about the datum line, although it does not lead to a unique choice of datum line. The mean value of the east component round any latitude circle must be zero at each instant. The integrals of the north component between the poles must be the same for all meridians. The north and east components must be derivable from the same potential function. There is, however, no test that may be applied to the vertical component. These tests are found to be satisfied more exactly when the night value is used as datum line than when the daily mean value is used, although in one or two cases it has been necessary to assume small non-zero night values.

6. The general comparison of the observatory data

6·1. *The purpose and method of comparison*

A direct comparison of the local-time daily variations of the three magnetic components at the observatories which operated during the Polar Year indicates the nature of the Sq field and its degree of dependence on longitude and latitude. A knowledge of the main features of this dependence is of value when the field has to be interpolated across areas where there are no observatories. A preliminary study of the magnetic data was therefore made for this purpose. Moreover, it was occasionally found that the data from a particular observatory did not show the general trends exhibited at nearby observatories, and it was considered desirable to examine such data more closely to discover, if possible, the cause of the discrepancy. It was then usually possible to decide whether the discrepancy represented a real feature of the Sq field, or whether it was due to incomplete elimination of extraneous effects or simply to computational errors in the reduction of the data.

The horizontal components used in the comparison were the geographic north (X) and east (Y) components. There is evidence that the Sq field is greatly controlled by the earth's magnetic field, and it might therefore seem that the magnetic components (H, D) would be more suitable for this purpose. There is, however, no satisfactory co-ordinate system which could be used with these components in deriving the magnetic potential. The geomagnetic north and east components would correspond to a possible co-ordinate system, but would have the disadvantage that points on the same geomagnetic meridian are not at the same local time.

It is convenient to group the observatories geographically into three sectors, each divided into northern and southern regions: sector A—North and South America; sector B—Europe and Africa; sector C—East Asia and Australasia. The grouping into sectors and regions in this way is justified by the fact that within each sector the variations show fairly regular changes with latitude, and that characteristic features peculiar to only one region are also found. The boundaries between the regions are not, however, precisely defined, since there are observatories (e.g. Alibag) which have variations that are transitional between those of two regions.

The range, type and phase are all used to compare the variations. The type is indicated by such principal features as the number, relative importance and breadth of the maxima and minima, and the changes of the field at dawn, dusk and during the night. The phases are compared by examining the local times of occurrence of these features. Many of these parameters were plotted for each season against the geographic and magnetic latitudes (see § 4·2) of the observatories; the graphs proved to be very useful in studying the distribution of the field and provided a basis for the map of the lines of corresponding latitude (see §§ 2·1 and 7·2). Some of these graphs are given below in figures 5 and 6.

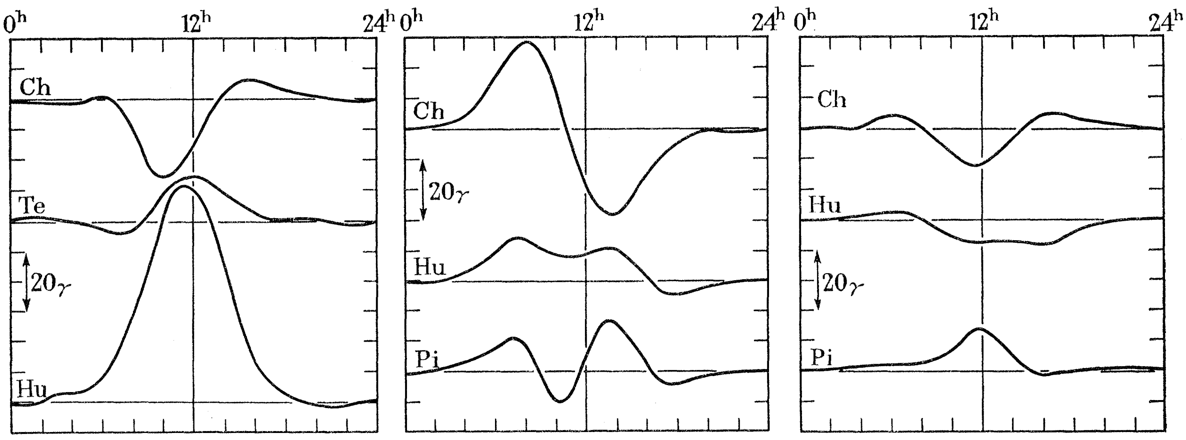


FIGURE 4. The main types of Sq variations. (*a*) Sq(*X*) at Cheltenham, Teoloyucan and Huancayo. (*b*) Sq(*Y*) at Cheltenham, Huancayo and Pilar. (*c*) Sq(*Z*) at Cheltenham, Huancayo and Pilar. Examples are taken from reduced data for J-months of 1933.

6·2. *The principal types of daily variation*

Graphs of the daily variations of *X*, *Y*, *Z* during each of the seasons J, E, D were plotted according to local time, and grouped into the three longitudinal sectors A, B, C defined above. Within each group the curves were arranged in order of decreasing latitude of the observatories; their spacing was however chosen simply with the view to separating consecutive curves as far as possible. The curves for all 44 observatories for the three seasons are given by Wilkins (1951).

The curves for each component exhibit the well-known features of the Sq variations. They are divisible into three main types as illustrated in figure 4; within each division there are, however, differences of range and phase, and minor differences of form. The types of the *X*-curves are temperate (e.g. Cheltenham), equatorial (e.g. Huancayo), and transitional (e.g. Teoloyucan), while those of the *Y*- and *Z*-curves are northern (e.g. Cheltenham), southern (e.g. Pilar), and transitional (e.g. Huancayo). The Sq(*X*) variation of temperate type has a principal minimum roughly at noon; that of the equatorial type has a maximum at about this time. The Sq(*Y*) variation of northern type has a morning maximum and an afternoon minimum; these are reversed in the southern type. The Sq(*Z*) variations of northern type has a midday minimum; that of southern type has a midday maximum. The transitional curves have properties characteristic of one, both or neither of the other two types.

The transition of Sq(X) from temperate to equatorial type is often not simply one of a reduction of amplitude of the minimum followed by a building up of the maximum. Frequently the time of minimum becomes earlier or later, depending on the hemisphere and season, and its amplitude decreases; at the same time a maximum builds up and moves towards midday. The transitions of Sq(Y) and Sq(Z) are less clear because of the small number of observatories in the equatorial regions, but it seems unlikely that there are any places where the variations are practically zero, even in the E-months. The distribution of Sq(Z) is less regular than for Sq(H), and sometimes quite marked differences are found between nearby observatories.

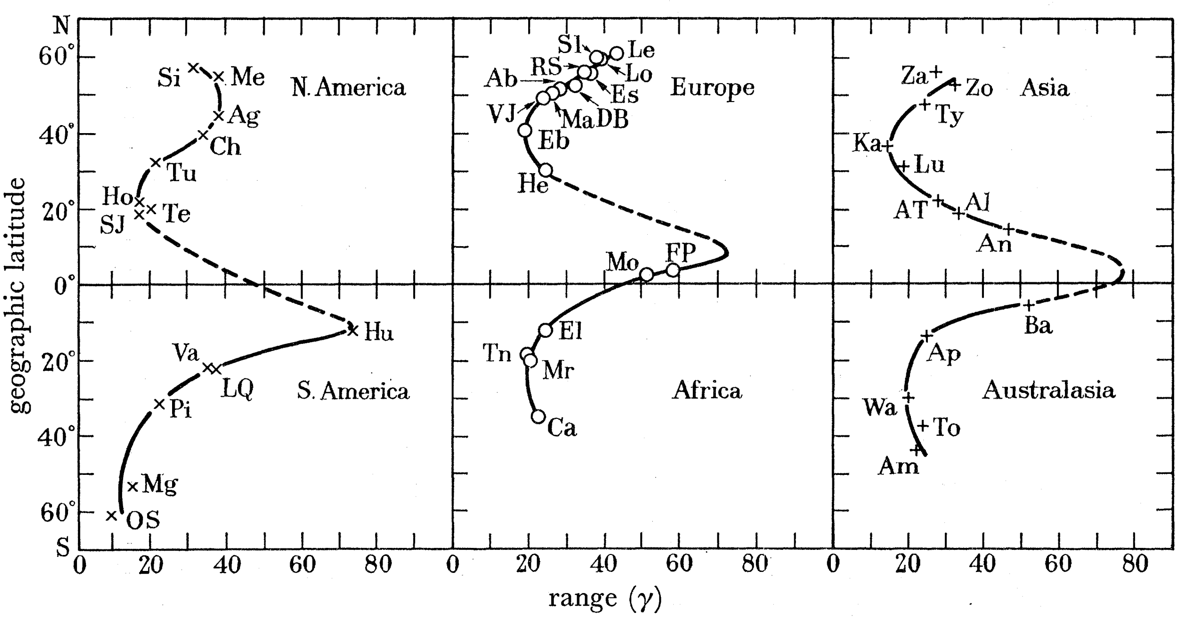


FIGURE 5. The ranges of Sq(X) during the J-months of 1933 plotted against geographic latitude ϕ for the American, Europe–African and Asia–Australasian sectors separately.

6·3. *The Sq field in equatorial regions*

The interpolation of the field components and the determination of the potential is not completely satisfactory in the equatorial regions since the number of suitably placed observatories was small during the Polar Year. The first step in the interpolation is to find as accurately as possible the position of the equatorial line of corresponding latitude; this is taken to be the same as the line along which the amplitude of Sq(X), or of Sq(H) since Sq(Y) is comparatively small, is a maximum. The range R_X of Sq(X) for the J-months of 1933 is plotted in figure 5 against geographic latitude ϕ for each of the three sectorial groups of observatories. The scatter is small within each group and there is clear evidence of the differences between corresponding latitudes for the three sectors, which roughly centre on longitudes 120° E, 30° E and 300° E. (In fact, by using corresponding latitudes taken from the map in figure 7, it has been possible to use ranges of observatories in one sector to supplement those of another.) Though these three curves are insufficient to establish

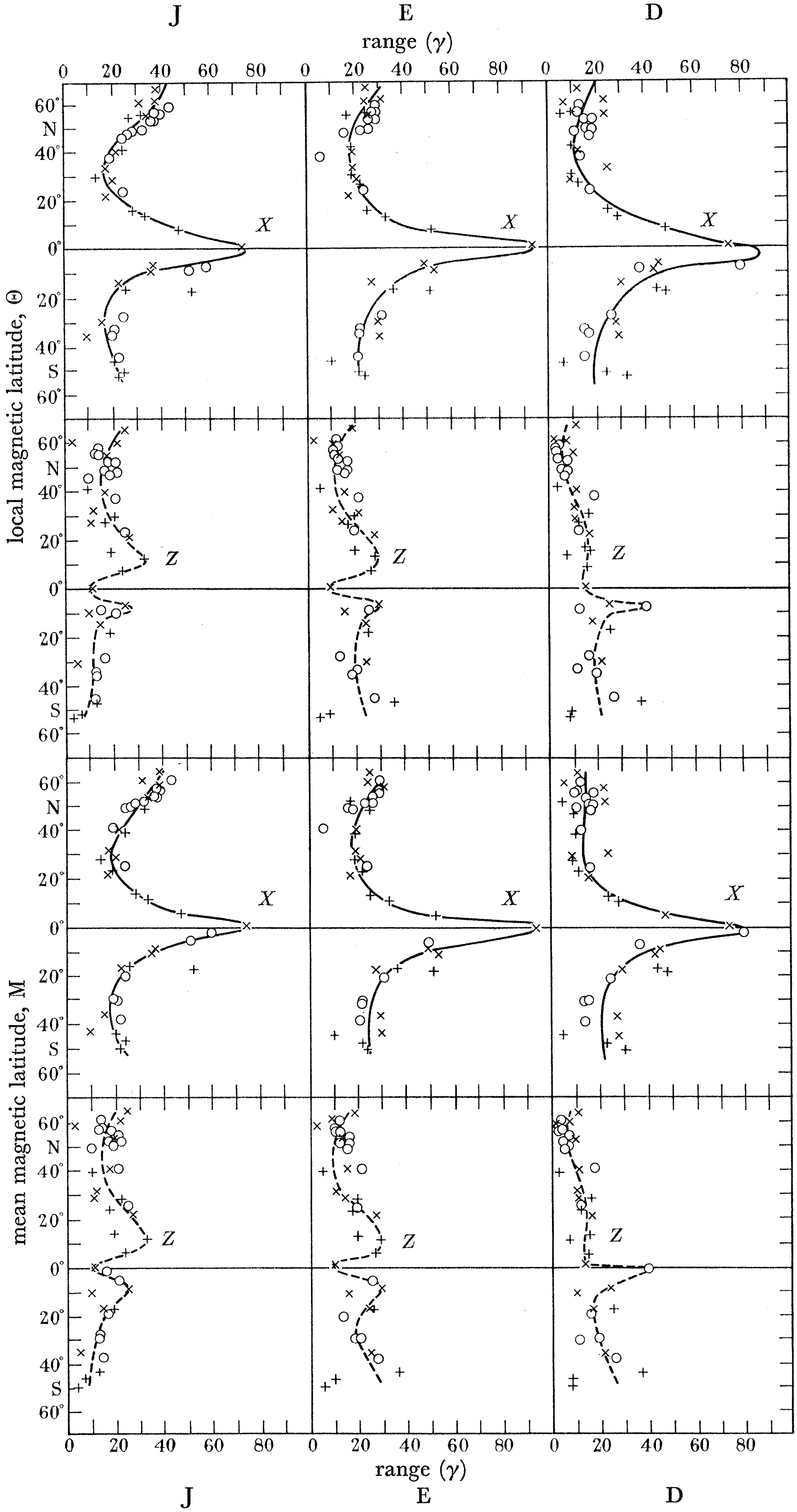


FIGURE 6. The ranges of Sq(X) and Sq(Z) for each of the three seasons J, E, D of 1932–3 plotted against local magnetic latitude Θ and the mean magnetic latitude M. American stations denoted by ×, Europe–African by ○ and Asia–Australasian by +.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

the position or intensity of the equatorial peak in R_X, they show clearly that the peak is much further south in the American sector than it is in the European–African one. The interpolation in figure 5 suggests that the peak in the Asian sector is only a few degrees from the geographic equator, i.e. it is several degrees south of both the geomagnetic and the dip equators; further the peaks are of comparable magnitude in all three sectors.

The ranges R_X and R_Z are also plotted in figure 6 against both local magnetic latitude Θ and mean magnetic latitude M for each of the three seasons of the Polar Year. Though curves can be found which fit most of the points fairly well, there are clearly one or two decidedly discordant points (e.g. Batavia) in low latitudes and some scatter in mid-latitudes. In the absence of conclusive observational evidence for the position of the line of maximum range the line shown on the map in figure 7 was finally adopted (Wilkins 1951); a slightly

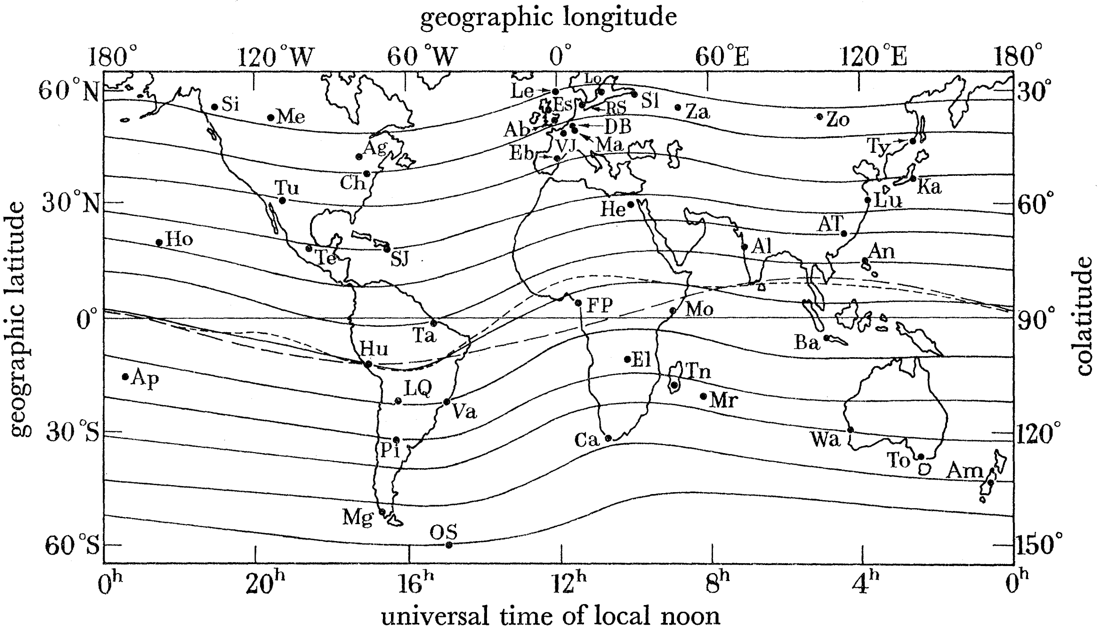


FIGURE 7. Map of corresponding latitudes of Sq for the J-months of 1933 showing observatories from which data were used. – – – dip equator, ——— geomagnetic equator.

earlier version of this map in a Mercator projection was published separately (Price & Wilkins 1951). This line was chosen because it was reasonably consistent with the equatorial observations then available to us, and with the lines of corresponding latitude that satisfied the observations in higher latitudes; further, its use led to the minimum discordances when the potential was deduced from the north and east components.

The study of the Polar Year for all three components indicated several other interesting features of the Sq field in the equatorial regions. Thus the variations of the horizontal and vertical components of the field at Huancayo on 5, 6, 7 March 1933 (see figure 3 for ΔH) suggest that the latitude of the electrojet responsible for the large variations in the horizontal component varies from day to day. There also appears to be a variation of the latitude of the electrojet with the season, its movement being in the opposite direction to that of the sun. It should, however, be noted that our estimate of the position of the equatorial electrojet depends partly on extrapolation from low and mid-latitudes, and this apparent variation of its position may be due more to the redistribution of currents in these latitudes than to the motion of the jet itself. If such day-to-day and seasonal motions do occur the graphs of figure 5, showing the ranges of Sq(X) plotted against geographic latitude will tend to show lower, but broader, peaks than would similar curves for individual days; they therefore cannot be used directly to estimate the distribution of intensity across the electrojet.

From our study it was clear, however, that simultaneous observations of all three components at several adjacent points of the same longitude are necessary for a reliable determination of the day-to-day changes of the position, height and distribution of intensity of the electrojet; further, many more observations are required to establish definitely the extent of the changes of position and intensity with longitude, season and solar activity.

More recent observations of the daily variations near the dip equator (Romana & Cardus 1955; Onwumechilli 1959; Forbush & Casaverda 1961) suggest that the maximum in R_X is somewhat sharper than that indicated in figure 6, and occurs near the dip equator in Africa as well as in South America; also that the two maxima in R_Z are higher and sharper than those in figure 6 and occur within 3 or 4° of the dip equator. The Polar Year data were inadequate to localize and evaluate these maxima very accurately so that our interpolation of the Sq field in the equatorial region may not be completely satisfactory. However, this is unlikely to affect seriously our picture of the Sq field for the J-months of 1933 except near the magnetic equator, and there would be no justification at this stage for attempting to modify the picture in equatorial regions. A new and comprehensive analysis of the Sq field with special attention to the equatorial regions is clearly desirable, and is now being undertaken by one of us using the data collected during the International Geophysical Year. This will give a picture of the field during a period of high magnetic activity. For comparison with the Polar Year a study of the field during the International Year of the Quiet Sun will be valuable, and it is very desirable that further temporary magnetic stations should be set up during that period, especially in the Pacific and mid-Atlantic Oceans where observations are scarce.

6·4. *The representative current-systems*

The general features of the Sq variations show that the ionospheric currents which would produce the Sq field form two vortex systems, one in the northern and the other in the southern part of the daylight hemisphere, the distribution being approximately constant as viewed from the sun. The current lines are roughly the same as the equipotentials of the external field; these, in turn, are similar to the equipotentials of the total field (see, for example, figure 9). The measurement of the field from a night value makes it unnecessary to postulate the existence in the night hemisphere of the similar (but weaker) vortices that are required when the field is measured from the mean of the day. The differences between the variations for the three sectors indicate that the intensity and shape of the representative current system vary with universal time. The latitudes of the foci of the current systems are those at which Sq(X) is of transitional type and Sq(Y) has a large range. The latitude of equatorial electrojet is taken to be that at which Sq(X) has its maximum range; Sq(Z) and Sq(Y) are of transitional type near this latitude.

The estimated latitudes of the principal features of the current system are shown in table 15, which has been obtained mainly by direct examination of the types and ranges of the variations, but also partly from the more detailed analysis that is described in §7. The table shows that, in addition to the changes with universal time, there are seasonal changes of latitude of the foci of the current vortices and it also suggests a possible seasonal motion of the equatorial electrojet in the American sector, but this is much less certain. The nature of the Sq(X) and Sq(Z) variations at Huancayo and elsewhere tend to suggest seasonal displacements, as well as possible day-to-day variability of position, but the observations in equatorial regions during 1933 are really not adequate to decide this point.

Table 15. Latitudes of the principal features of the representative current systems for Sq during the Polar Year, 1932–3

season	15^h to 18^h U.T. America	8^h to 12^h U.T. Europe–Africa	2^h to 5^h U.T. Asia-Australasia
focus of northern vortex			
J-months	25° ± 3° N.	35° ± 3° N.	30° ± 3° N.
E-months	25° ± 3° N.	40° ± 3° N.	35° ± 3° N.
D-months	20° ± 3° N.	35° ± 3° N.	35° ± 3° N.
electrojet at noon			
J-months	13° ± 2° S.	8° ± 3° N.	5° ± 5° N.
E-months	12° ± 2° S.	8° ± 3° N.	5° ± 5° N.
D-months	11° ± 2° S.	8° ± 3° N.	5° ± 5° N.
focus of southern vortex			
J-months	45° ± 5° S.	33° ± 3° S.	35° ± 5° S.
E-months	45° ± 5° S.	28° ± 3° S.	30° ± 3° S.
D-months	45° ± 5° S.	28° ± 3° S.	30° ± 3° S.

Even if our estimates of the latitude of the electrojet are not quite correct our general picture of the Sq field is not seriously affected. It should be noted that there is no reason to assume the line of the electrojet coincides (even at noon) with the boundary line between the two vortex systems. The boundary line certainly varies in position with the season, but this does not necessarily mean that the electrojet moves.

There are significant differences in the Sq field between the northern and southern regions of the same sector even during the E-months. Differences of range during the D- and J-months are to be expected because of the change in the distribution of conductivity of the ionosphere, but there are other important differences such as those in the phases of the corresponding maxima and minima of Sq(Y). These phase differences are largest in the J-months, and they imply that the current vortex in the southern hemisphere then lags by about 2^h behind that of the northern hemisphere. Further the marked early morning maxima of Sq(Y) for many southern observatories suggests that the northern system penetrates well into the southern hemipshere at this time; this is confirmed by an examination of Sq(X) and Sq(Z). This implies that the boundary line between the northern and southern current systems does not coincide with the magnetic equator as suggested by Matsushita (1960). During the D-months the northern system lags slightly behind the southern system, and the penetration of the southern system into the northern hemisphere is indicated in the Asian region, where there is a small minimum of Sq(Y) in the morning. These effects are, however, less marked than those during the J-months. During the E-months, when some symmetry about the equator might have been expected, there is still a definite lag of the southern current system behind the northern one.

The seasonal motion of the focus of the northern current system over the American sector in the first column of table 15 agrees well with that derived by Matsushita (1960) from a study of data from 10 North American stations during the I.G.Y. This motion is—as pointed out by Matsushita—in the sense opposite to that indicated by the diagrams of the current systems obtained by Bartels for 1902 (Chapman & Bartels 1940, p. 229). In the Asian region, however, the table indicates a seasonal motion of the northern focus which agrees with Bartel's diagrams, while in the Europe–African sector the focus appears to be furthest south during the E-months. This suggests that the average positions of the ionospheric current systems at different times of the year are not controlled solely by the position, north or south, of the sun relative to the magnetic equator.

6·5. *Local features of the Sq field*

The data from some observatories show special features of the daily variations and these suggest that the Sq field is subject to local influences as well as to world-wide causes. The existence of such local effects must be taken into account when attempting to interpolate the field over the earth. Local effects appear to be present in all three components, but are particularly noticeable in Sq(Z). Some examples are (i) the two early morning maxima of Sq(Z) appearing in all seasons at Capetown but at no other observatory, (ii) the notably large range of Sq(Z) at Watheroo for the D- and E-months, and at Honolulu for the E- and J-months, and (iii) the extremely small range of Sq(Z) at Agincourt at all times. Large ranges of Sq(Z), as well as of Sq(X), were also recorded at Fernando Po during D-months, and this suggests that during those months the equatorial electrojet was not far north of this station. On the other hand the values were not exceptional during the J-months, and this could be regarded as supporting the previous suggestion that the electrojet might have a seasonal movement. There were, however, some large night changes recorded at Fernando Po, unlike those at any other Polar Year station; this raises some doubts about the reliability of the data recorded there, but recent observations at Neifang (Romana & Cardus 1955) give similar night changes.

7. The Sq field during the J-months of 1933

7·1. *Summary of procedure*

The methods used for the interpolation over the earth of the observed variations of the Sq field components and the subsequent determination of the potential have been described in part I, in §§2·1 and 2·22, respectively. In this section, therefore, we merely summarize the procedure adopted and draw attention to a few points that are peculiar to this particular investigation.

The basic data consisted of the seasonal mean values of Sq(X), Sq(Y), Sq(Z) for each hour of local mean time (L.T.) at 44 observatories (see §4·2) for the J-months (May to August) of 1933. These values were derived (see §4·3) from the mean variations on the international quiet days and were measured from a night value (see §5·5); the values for 0^h and 24^h L.T. were equal (see §5·4) and were usually zero.

The first step was to obtain by graphical interpolation the values of Sq(X), Sq(Y), Sq(Z) at a world-wide network of points for the epochs 4^h, 8^h, 12^h and 16^h U.T. for the J-months of 1933. (The reasons for the choice of this season and these times have been given in §4·1.) The next step was to determine for each of these epochs by the method of residuals the values of a potential function Sq(Ω) that was consistent with the values of Sq(X) and Sq(Y). The final step, which is described below in § 8, was to use the values of Sq(Ω) and Sq(Z) to determine the separate contributions to the potential of the field sources of external and internal origin.

For each epoch and each component the values of the field were estimated on each hour-meridian (i.e. at intervals of 15° in longitude from the Greenwich meridian) by assuming that the local-time variations of the field are approximately constant along the lines of corresponding latitude for Sq for up to about 3^h from each observatory (see figure 7). Smooth curves were drawn through these primary estimates in such a way as to give smooth variations of the field along the mesh-circles (i.e. latitude circles at intervals of 10°). From the curves of Sq(X) along the hour-meridians and of Sq(Y) along the mesh-circles the potential differences between the points of a coarse mesh at intervals of 20° of colatitude and 2^h of longitude were formed, and the residuals for each mesh-circuit were calculated. (The residual is the sum of the potential differences, taken with appropriate signs, around the circuit. It should be zero if the force components are derivable from a potential function.) The estimates of the field, and hence of the potential differences, were adjusted until all residuals were zero; as far as possible all adjustments were made in such a way that the interpolated variations of the field were compatible with the basic data. The process was then repeated for a finer mesh at intervals of 10° of colatitude and 1^h of longitude. The isomagnetic lines and equipotentials for each epoch are shown in figures 8 to 15, in which a simple rectangular 'projection' has been used.

The interpolation of the field components took into account the results of general investigations (described in § 6) of the nature of the Sq field, particularly in the equatorial regions. On the other hand, the attempt to fit a potential to both Sq(X) and Sq(Y) simultaneously drew attention to some interesting features of the field and, in particular, suggested the use of the night-value as the datum from which the Sq field should be measured.

No account was taken of observations in the polar regions; the field was merely extrapolated smoothly to zero at the poles. The isomagnetic lines and equipotentials were drawn separately on a polar projection to verify that this extrapolation was reasonable; the rectangular projection, although suitable for the rest of the world, was inappropriate in the polar regions.

7·2. *The map of the lines of corresponding latitude*

The map of the lines of corresponding latitude for Sq that was used in the interpolation of the field components is given in figure 7; it was based in the first instance on a few special loci such as the locus of maximum range of Sq(X) and the loci of transition of type of Sq(X). The latter are nearly but not exactly the same as the loci of maximum range of Sq(Y). Sq(X) proved the most useful for indicating the general features of the map; Sq(Y) and Sq(Z) provided additional information and important checks. The difficulties in establishing the locus of maximum range of Sq(X) have been discussed above in §§ 6·3 and 6·4; it should also be noted that in the equatorial regions it appeared impossible to find any single locus that was appropriate to all three components.

The two loci of transitional Sq(X), which correspond roughly to the paths of the foci of the current vortices, were first sketched in by estimating their average latitudes for each of the three sectors A, B, and C (§ 6·1). Their positions were then determined more accurately by considering the nature of all three components at nearby observatories during the interval 11^h to 13^h L.T. At a focus, Sq(X) and Sq(Y) should ideally be zero and Sq(Z) should have a minimum. These loci were found to run roughly parallel to the equatorial electrojet; the variation in latitude as they circle the earth is rather greater (for the J-months) than that estimated by Hasegawa (1960) for the mean of the two solstitial seasons.

The detailed interpolation of the field revealed some deficiencies of the preliminary map of corresponding latitudes, and indicated some adjustments which would improve it. A check on the lines of corresponding latitude was afforded by using them to supplement the latitude-range graphs of figure 5, with data from observatories lying up to 90° or more from the central meridian of each group.

In the northern hemisphere the lines of corresponding latitude show smaller departures from the geographic latitude circles than those in the southern hemisphere. There are fewer observatories in the southern hemisphere. Hence the field is determined less accurately there; but the data suffice to show that this feature is quite real. Although the lines are intended to give the directions along which the changes in the Sq field are least, appreciable changes sometimes occur even along these directions. This must be allowed for in interpolating the field. Also there are some stations where one particular component (e.g. Sq(Z) at Capetown) is anomalous, or again different components within a region may suggest slightly different lines. Nevertheless, it is found possible, by using these lines, to fit a reasonably smooth Sq field to the observatory data without serious discrepancies, whereas this is not possible if the circles of geographic latitude are similarly used (Hasegawa 1936).

7·3. *Difficulties encountered in deducing the potential*

It is possible to find a potential that fits satisfactorily in all but a few areas the charts of Sq(X) and Sq(Y) that are given in figures 8, 10, 12 and 14. The main difficulties occur in the equatorial and southern regions near the local noon meridian. The primary points of Sq(X) generally lie on a fairly smooth curve, but those of Sq(Y) may have a considerable scatter. This scatter is least at 4^h U.T., but even at this time the adopted potential field departs from original estimates of Sq(X) and Sq(Y) by up to about 5γ. At 8^h and 12^h U.T. the Sq(X) curves are just consistent with the limits of scatter of the Sq(Y) points, although it is not possible to take into account the various weights of these points. At 16^h U.T. Sq(X) is given the greater weight in the determination of the potential, Sq(Y) at Huancayo, La Quiaca and Vassouras being largely neglected; the primary estimates of Sq(Y) for points between Huancayo and Fernando Po differ by as much as 30γ.

It is difficult in some cases to interpolate Sq(Y) in such a way that, as required for the existence of a potential, the mean value along each mesh-circle between 0 and 30° S. is zero. It is easier to satisfy this condition if it is assumed that the field at some southern observatories has a non-zero night value, but even then it is necessary to depart from the observations. The determination of Sq(Y) in these equatorial and southern latitudes is the least satisfactory part of the analysis.

The principal cases where the adopted potential field apparently differs systematically from the observations are as follows:

At 4^h U.T. in Australasia the differences are as much as 5γ; there are also smaller differences in northern latitudes.

At 8^h U.T. the potential field gives smaller values of Sq(Y) than the observations, by between 2 and 4γ, near the meridian 60° W in southern latitudes; it has not been possible to take into account the large variation of Sq(Y) at Batavia.

At 12^h and 16^h U.T. there is some difficulty in interpolating across the North Atlantic Ocean, but this does not appear to be serious.

At 16^h U.T. it has been necessary to depart from the observations of Sq(X) by 2 to 5γ in the region of 120° to 90° W and 30° to 50° N.

At some observatories the assumed departures from the observations are larger than those that can be reasonably attributed to observational error, but it seems likely that they are due to local irregularities, and there is no conclusive evidence for the existence of a non-potential part of the field.

7·4. *The principal features of the field charts*

The isomagnetic lines of the Sq(X), Sq(Y), Sq(Z), measured as deviations from a local night value, are shown in figures 8, 10, 12 and 14 for the epochs 4^h, 8^h, 12^h and 16^h U.T. of the J-months of 1933. In each chart the local noon meridian is placed at the centre. Thus the charts show the field as viewed from the sun. The field over the Pacific Ocean has been estimated by interpolation over considerable distances. It is therefore subject to some uncertainty; but the charts have been drawn only for those epochs for which this area is not of primary importance.

The charts for the vertical component are the most complex; this might be expected, since any induced earth currents (including currents induced in the highly conducting oceans) have a much greater proportional effect on the vertical components than on the horizontal ones. No obvious correlations of the field patterns with the distribution of land and sea are, however, discernible.

The equipotentials of the total Sq field for the epochs 4^h, 8^h, 12^h and 16^h U.T. of the J-months of 1933 are shown in figures 9 (Ω), 11 (Ω), 13 (Ω) and 15 (Ω), in each of which the local noon meridian is placed at the centre. Apart from the exceptions mentioned in §7·3, the charts of Sq(Ω) are consistent with those of Sq(X) and Sq(Y).

The equipotentials may be expected to be similar in form to the current lines of the equivalent electric current system. It must be noted, however, that the measurement of the variations from their night values and the choice of the arbitrary constant in the potential have necessarily resulted in the absence of any significant features in the night hemisphere of each chart. With the adopted choice of the arbitrary constant in the potential the values of the maxima and minima at the foci probably give an indication of the relative strengths of the northern and southern current systems, and of their changes with universal time. It is clear that in the J-months the northern system is not only much more intense than the southern system, but also spreads into the southern hemisphere, particular in the morning and to a lesser extent in the evening. The morning effect appears to be associated with an eastward displacement of the southern focus with respect to the northern focus. Both northern and southern systems show changes of intensity, latitude, phase and shape with universal time, and it seems that these changes indicate that the current systems tend to move north and south with the magnetic equators.

The changes of inclination of the equipotentials to the geographic latitude circles near the equatorial point of maximum field intensity are of particular interest. At 12^h U.T.

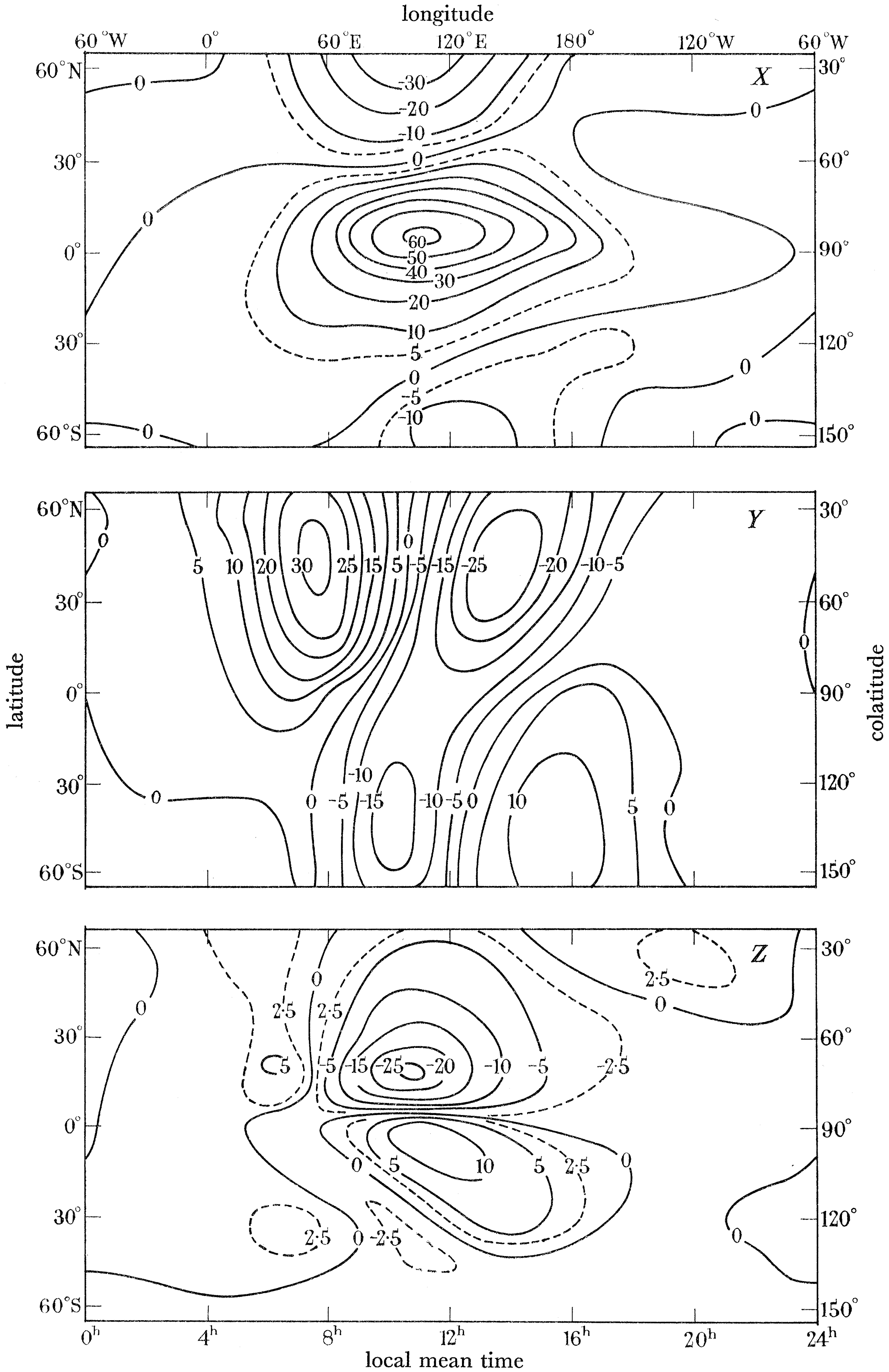


FIGURE 8. Isomagnetic lines of Sq(*X*), Sq(*Y*) and Sq(*Z*) at 4^h U.T., J-months, 1933. Unit = 1γ.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

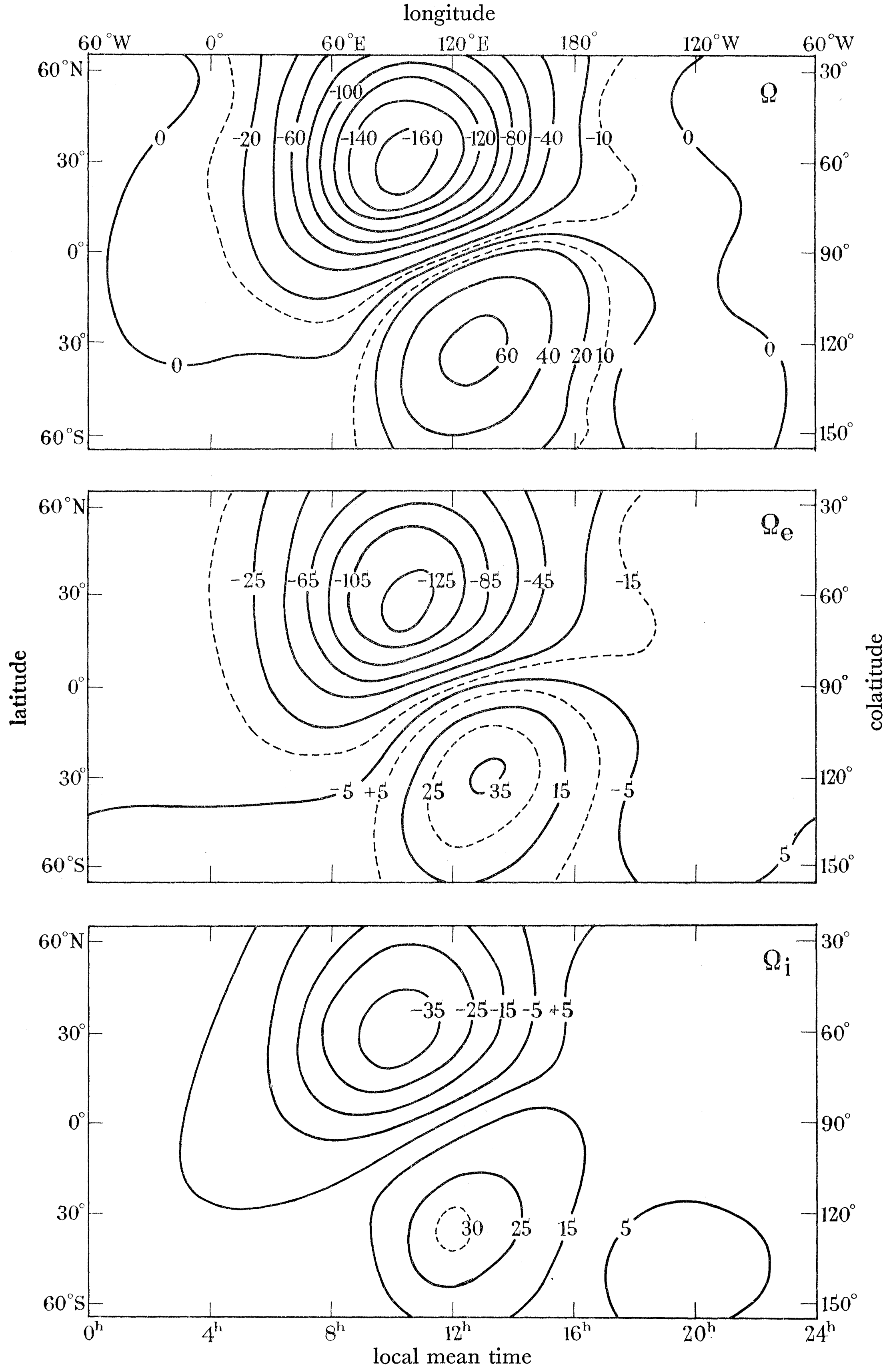


FIGURE 9. Equipotentials of Sq(Ω), Sq(Ω_e) and Sq(Ω_i) at 4^h U.T., J-months, 1933. Unit = 10^3 gilberts.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

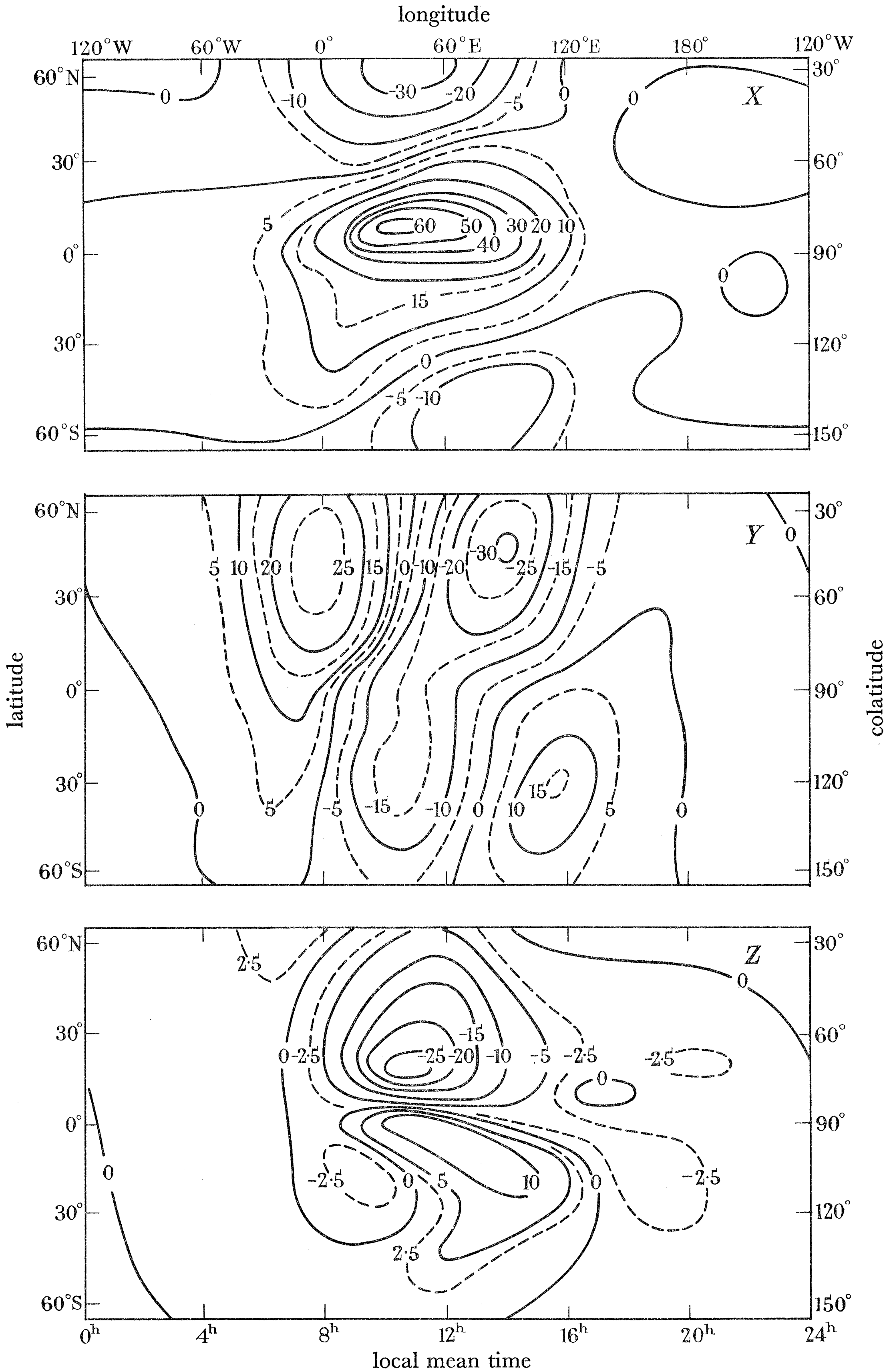


FIGURE 10. Isomagnetic lines of Sq(X), Sq(Y) and Sq(Z) at 8^h U.T., J-months, 1933. Unit = 1γ.

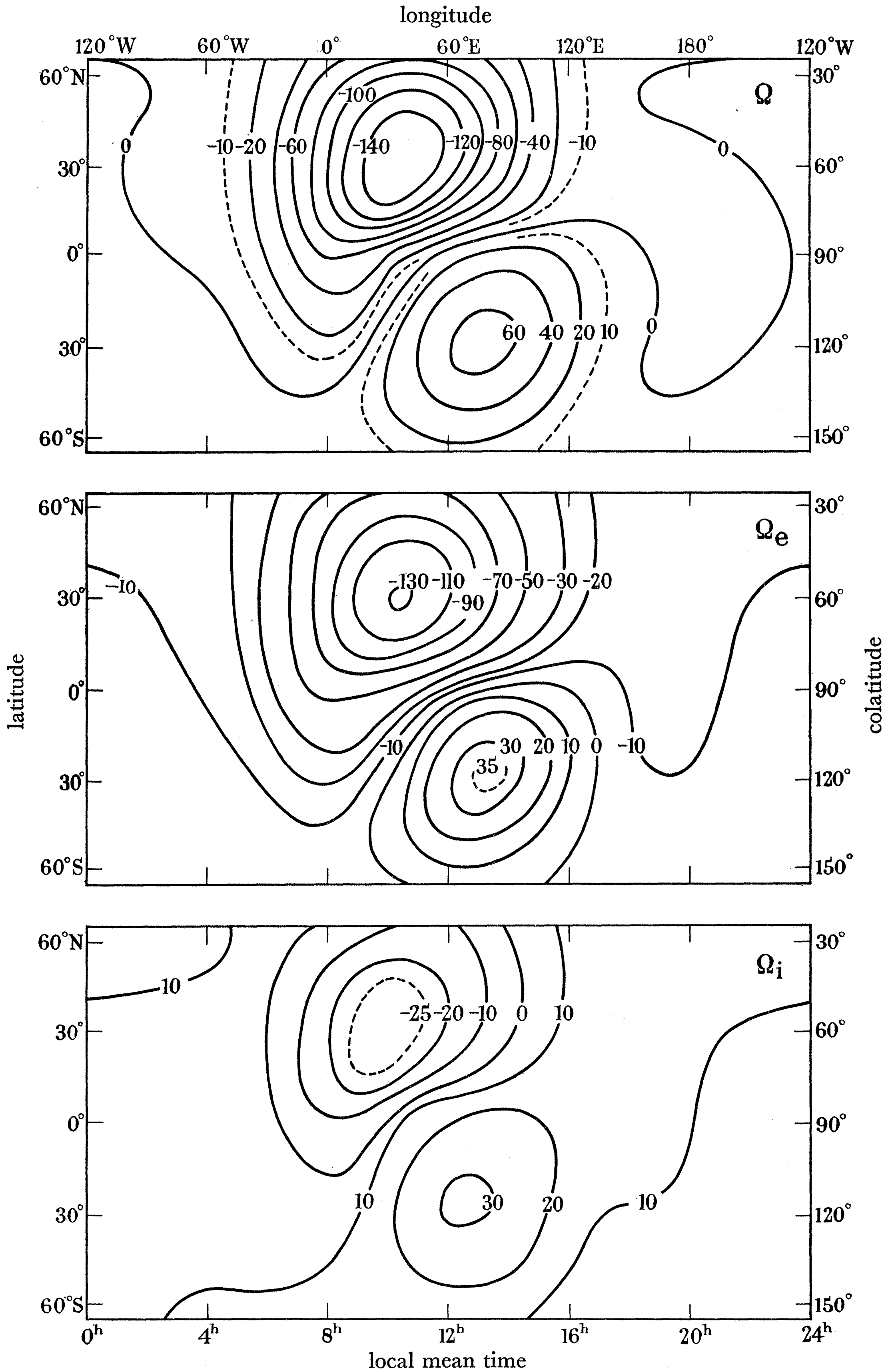


FIGURE 11. Equipotentials of $Sq(\Omega)$, $Sq(\Omega_e)$ and $Sq(\Omega_i)$ at 8^h U.T., J-months, 1933. Unit = 10^3 gilberts.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

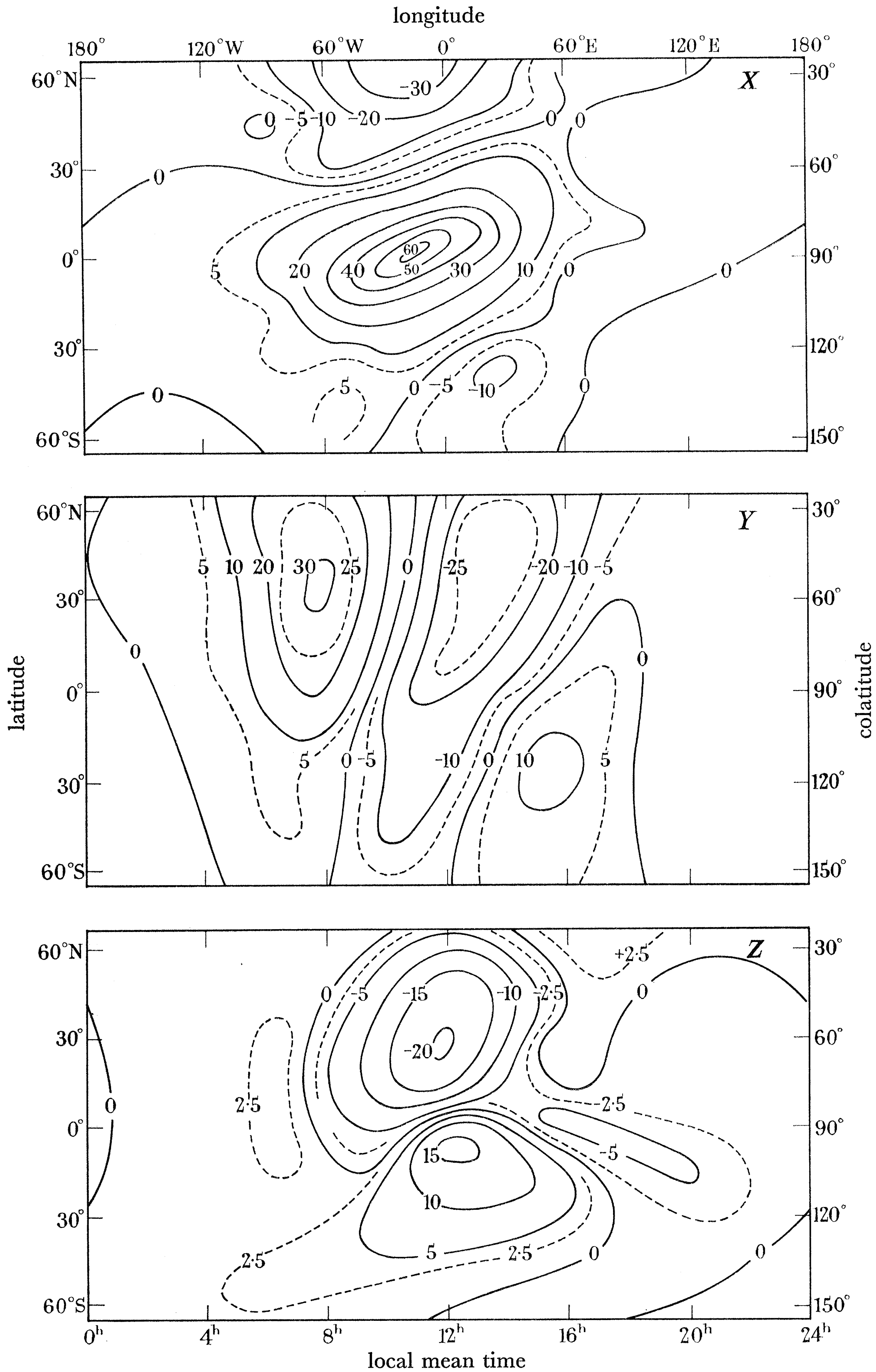


FIGURE 12. Isomagnetic lines of Sq(X), Sq(Y) and Sq(Z) at 12^h U.T., J-months, 1933. Unit = 1γ.

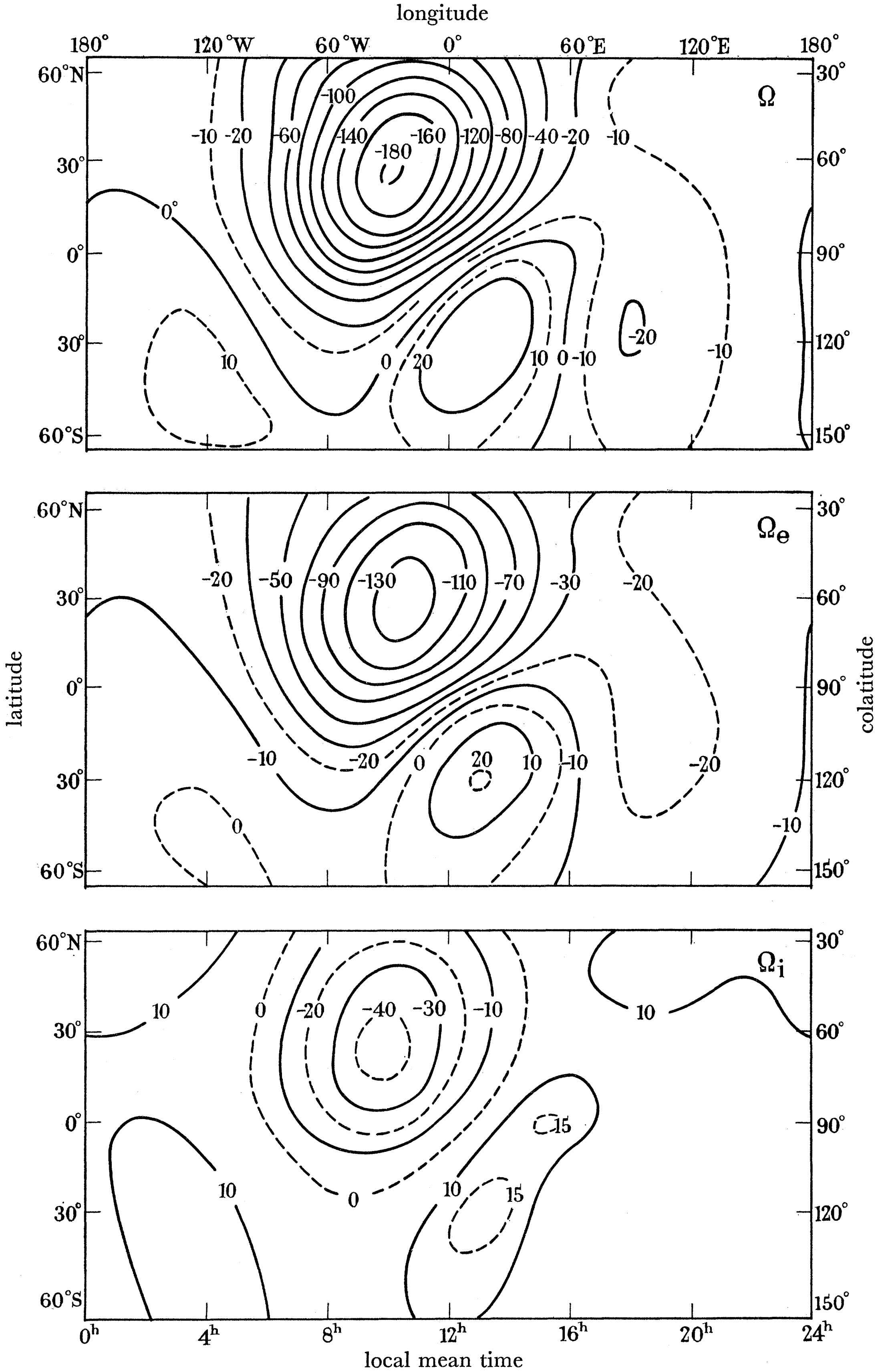


FIGURE 13. Equipotentials of Sq(Ω), Sq(Ω_e) and Sq(Ω_i) at 12^h U.T., J-months, 1933. Unit = 10^3 gilberts.

12-3

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

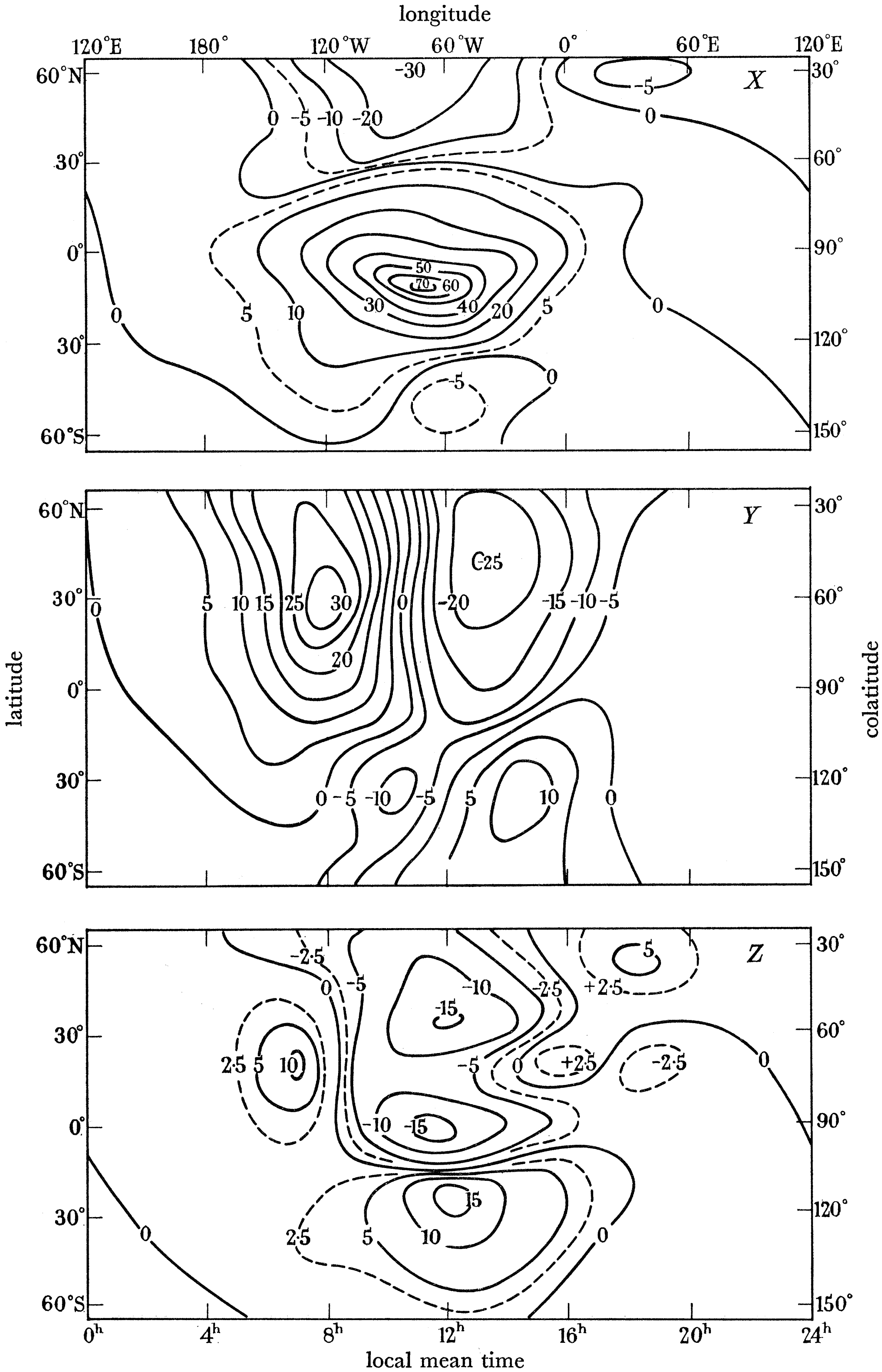


FIGURE 14. Isomagnetic lines of Sq(X), Sq(Y) and Sq(Z) at 16^h U.T., J-months, 1933. Unit = 1γ.

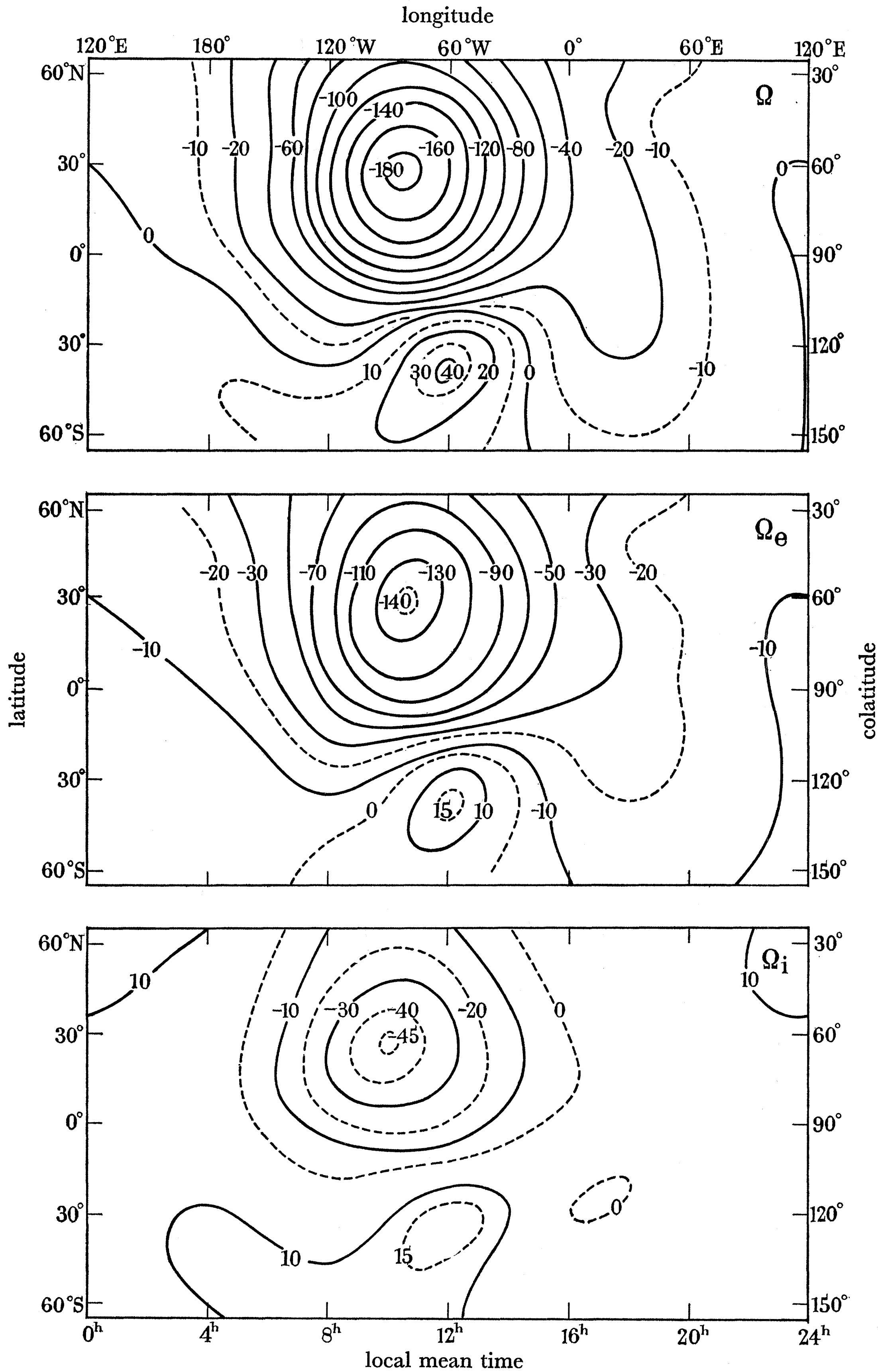


FIGURE 15. Equipotentials of Sq(Ω), Sq(Ω_e) and Sq(Ω_i) at 16^h U.T., J-months, 1933. Unit = 10^3 gilberts.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

the large inclination appears to be associated with that of the magnetic equator. At 16^h U.T. the true inclination of the lines is not certain, since the east component at Huancayo could not be fully taken into account in obtaining the potential; it is possible that there should be a slight inclination in the opposite direction to that shown in figure 15 (Ω). The inclination at 4^h and 8^h U.T. appears to be real, suggesting that the current lines are not parallel to the magnetic equator at all times; it may be noticed that the inclination of the isomagnetic lines of the vertical component at 8^h U.T. (see figure 10 (Z)) is not the same as that of the equipotentials.

Other features of the field are considered in § 8·2, where the results of the separation of the potential into external and internal parts are discussed.

8. The separation of the potential

8·1. *Summary of procedure*

The principal step in the separation of the potential (Ω) into parts of external (Ω_e) and internal (Ω_i) origin by the method of surface integrals (see § 3) was the evaluation of the 'scalar product'

$$(\Omega_e - \Omega_i)_A = \sum_N (\Omega + 2aZ)_N I_N^A$$

for each of a large number of points (A) of the network of points (Q_N) at which the potential and vertical components of force (Z) are known. (In this section we use Ω for Sq(Ω), etc.) The network comprised some 408 points lying at the intersections of the hour-meridians at intervals of 15° of longitude and small circles at intervals of 10° of colatitude, together with the two poles; the small contributions from the latter were added separately after the main summation and are not considered further in the following description of the arrangement of the calculations.

The field data, i.e. the values of $\Omega + 2aZ$ for each of the points for each epoch, were readily calculated by hand, use being made of a short table of $2aZ$, from the values of Ω and Z; when Z is measured in gammas, $2a$ is 12·74, as shown in § 3·2. These data and the values of the basic integrals I_N that are given in table 1 were punched on cards in a form suitable for input to the computer. (In any extended application of this method it would probably be desirable to punch Ω and Z separately and to form $\Omega + 2aZ$ in the computer.)

The field data for one epoch at a time were stored on the magnetic drum of the computer in such a way that the values for the subvectors for each meridian were in consecutive locations. The values of I_N^A for one colatitude (θ_A) at a time were stored similarly, although, to reduce the amount of storage space required, they were stored only for positive values of the longitude difference between A and N. The calculation of $(\Omega_e - \Omega_i)_A$ for a given longitude λ_A was carried out by summing the scalar products, for each meridian ($\lambda_N = 0, 1, 2, \ldots, 23$ in turn), of the subvectors for the field data and subvectors for the corresponding values of I_N^A. The subvectors of I_N^A were selected by forming the discriminant

$$D \equiv |\lambda_A - \lambda_N| - 12$$

and then using, instead of the actual longitude difference, $12 + D$ when $D \leqslant 0$, and $12 - D$ when $D > 0$. Once the field data and a set of values of I_N^A had been loaded, the computer formed and printed the values of $(\Omega_e - \Omega_i)_A$ for each of the 24 points ($\lambda_A = 0, 1, 2, \ldots, 23$) at that chosen colatitude (θ_A). Then the set of basic integrals for the next chosen colatitude was read in and the process repeated. At each stage appropriate sum checks were carried out to verify that the data were stored correctly on the drum.

The values of Ω_e and Ω_i were calculated manually from the equations

$$\Omega_e = \tfrac{1}{2}\{\Omega + (\Omega_e - \Omega_i)\}, \quad \Omega_i = \tfrac{1}{2}\{\Omega - (\Omega_e - \Omega_i)\},$$

and then plotted. In a more extensive application of the method it would be best to store Ω (or at least the values for colatitude θ_A), as well as $\Omega + 2aZ$, and to print the separate values of Ω_e and Ω_i.

8·2. *The comparison of the external and internal parts*

The results of the separation of the potential into parts of external and internal origin are shown in figures 9, 11, 13 and 15 (Ω_e and Ω_i, respectively) for the epochs 4^h, 8^h, 12^h and 16^h U.T. during the J-months of 1933. It is immediately apparent that, although the equipotentials for the total, external and internal fields are of the same general form, the external field is very much greater than the internal field and that there are phase differences between them. The principal features of the charts are summarized in table 16; the positions given are uncertain by about 2° in latitude and 15^m in local time.

The sum, without regard to sign, of the intensities of the northern (N) and southern (S) foci may be taken as a comparative measure of the total current flowing in the equivalent current system. The table shows that the mean ratio of the total intensities of the parts of external (Ω_e) and internal (Ω_i) origin is about 2·6. This agrees with the value of the amplitude ratios for the principal terms obtained from spherical harmonic analysis by Chapman (1919) for the years 1902 and 1905, but is larger than the value 2·1 obtained by Benkova (1940) for the J-months of 1933. The new value, however, may not be strictly comparable with the earlier values as the ratio is based on quite different measures.

The spherical harmonic analysis indicated phase differences of about −7° between the corresponding principal terms in the external and internal parts. These new charts also indicate similar differences; the foci of the internal part are displaced by about half an hour in longitude to the west of the foci of the external parts. There are also variations in phase, as shown by the changes in the local times at the foci, from one epoch to another.

The relative intensities of the northern and southern current systems at the various epochs cannot be compared confidently. This is because the night values of the external and internal parts, unlike those of the total potential, are not zero. As has been pointed out in § 3·1, any change in the arbitrary constant in the total potential leaves the internal part unchanged and changes the external part by the same amount. Hence, it is clearly not justifiable to alter the external and internal parts by arbitrary constants in order to make them satisfy the same condition as the total potential, and it is not obvious how a more suitable choice of the arbitrary constant of the total potential could be made. It seems reasonable to assume, however, that the total current flowing around each focus is proportional to the difference of potential between the focus (N or S) and the night hemisphere (n). Thus in table 16 the ratio $(N-n)/(S-n)$ may be supposed to represent the relative intensity of the northern and southern current systems. This ratio varies quite considerably from one epoch to the next, particularly for the internal part of the field. At 12^h U.T. the southern internal system is very weak, and moreover seems to be split into two parts; it

TABLE 16. PRINCIPAL FEATURES OF THE CHARTS OF THE POTENTIAL OF THE Sq FIELD FOR THE J-MONTHS OF 1933

	total potential (Ω)				external part (Ω_e)				internal part (Ω_i)			
epoch: universal time ...	4^h	8^h	12^h	16^h	4^h	8^h	12^h	16^h	4^h	8^h	12^h	16^h
intensity												
night value (n)	0	0	0	0	−6	−8	−9	−9	+6	+8	+9	+9
northern focus (N)	−169	−158	−180	−182	−130	−130	−138	−140	−38	−29	−43	−45
southern focus (S)	+66	+67	+38	+41	+37	+36	+21	+16	+31	+31	+17	+17
total ($S-N$)	235	225	218	223	167	166	159	156	69	60	60	62
ratio $-(N-n)/(S-n)$	2·6	2·4	4·7	4·4	2·9	2·8	4·3	5·2	1·8	1·6	6·5	6·7
latitude												
northern focus	30° N.	30° N.	28° N.	28° N.	30° N.	30° N.	30° N.	28° N.	30° N.	32° N.	25° N.	28° N.
maximum field	5° N.	10° N.	2° N.	13° S.	5° N.	5° N.	2° N.	13° S.	5° N.	5° N.	2° N.	13° S.
southern focus	32° S.	30° S.	27° S.	37° S.	30° S.	28° S.	30° S.	37° S.	35° S.	30° S.	30° S.	35° S.
local time	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
northern focus	10 30	10 15	10 15	10 20	10 45	10 30	10 30	10 45	10 15	10 00	09 45	10 00
maximum field	11 15	10 30	11 00	11 15	11 00	12 00	11 30	11 30	11 00	11 00	11 00	11 00
southern focus	12 45	13 15	13 15	11 45	13 15	13 30	13 00	12 00	12 00	12 45	13 00	11 45
	total intensity ($S-N$)				northern systems ($N-n$)				southern system ($S-n$)			
ratio of external and internal parts Ω_e/Ω_i	2·4	2·8	2·7	2·5	2·8	3·3	2·5	2·4	1·7	1·9	3·8	3·1

is tempting to ascribe these peculiar results to the effects of the induction of currents in the oceans—the spread of the northern system into the southern being, perhaps, accentuated by the influence of the Atlantic Ocean. The secondary southern focus may be due to a current system in the Indian Ocean. Further investigation is required, however, before such effects can be said to have been established.

9. Conclusions

The results of the application of the new methods for the interpolation and analysis of surface magnetic fields to the Sq field during the International Polar Year 1932–3 demonstrate the flexibility and value of the methods and have drawn attention to several interesting features of the field. We believe that the charts given in this paper represent the observations of the Sq field during the J-months of 1933 better than any earlier charts. Since we have measured the daily variations from a night value our charts differ significantly from earlier charts, but we believe that they correspond more closely to the field of the electric currents responsible for the variations. Further the methods that we have used for the interpolation of the field components, the determination of the potential, and the separation of this potential into parts of external and internal origin have enabled us to include local features of the field that have been smoothed out in the earlier spherical harmonic analyses.

The paucity of observations in equatorial regions during the period considered does mean, however, that the results may still not adequately represent the field of the equatorial electrojet. In particular it is possible that the maxima for Sq(X) and, correspondingly for Sq(Z), at about 11^{h} L.T. should be greater and sharper, and should occur closer to the dip equator. However, if there are day-to-day or month-to-month changes in the latitude of the equatorial electrojet, as we have suspected, then we would not expect our results, which are based on the mean of 20 quiet days, to show the same sharpness that observations based on a few days have suggested. The present charts show clearly that the changes with universal time in the latitude of the equatorial electrojet are closely associated with striking changes in the intensity and distribution of the northern and southern electric current systems. Further, the charts show that the main features of the field are overlaid by many local changes, thus demonstrating the complex character of the Sq field.

This work could be extended by similar analyses of the data for the E- and D-months of 1932–33. Also the data for the auroral-zone observatories have not yet been fully analyzed. Further, spherical harmonic analyses of the results here obtained for the J-months would provide a useful analytical representation of the principal features of the field. It is hoped that this description of the new method and the example of this application to the Sq field in 1932–33 will prove useful in similar studies of the observations made during the International Geophysical Year 1957–8.

This paper is based on work which we started at Imperial College in 1949 and part of it was presented in 1951 by one of us (G. A. W.) as a thesis for the degree of Ph.D. of London University. Its completion and publication is in great measure due to the continued encouragement of Professor S. Chapman, F.R.S., and we are grateful to him for much valuable advice in its presentation. We wish to thank the Librarian of the Meteorological Office and

Dr V. Laursen, who was secretary of the Commission for the Liquidation of the Polar Year, for their help in obtaining much of the data.

The computations involved in separating the potential were carried out, by kind permission of the Astronomer Royal, on the I.C.T. 1201 computer in H.M. Nautical Almanac Office (Royal Greenwich Observatory, Herstmonceux Castle); Miss B. M. Harris assisted with these computations.

References

Benkova, N. P. 1940 *Terr. Magn. Atmos. Elect.* **45**, 425.
Chapman, S. 1919 *Phil. Trans.* A, **218**, 1.
Chapman, S. & Bartels, J. 1940 *Geomagnetism.* Oxford University Press.
Chapman, S. & Price, A. T. 1928 *Proc. Roy Soc.* A, **119**, 182.
Davids, N. 1944 *Terr. Magn. Atmos. Elect.* **49**, 239.
Forbush, S. E. & Casaverda, M. 1961 *Carnegie Inst. of Washington Publ.* no. 620.
Hasegawa, M. 1936 *Proc. Imp. Acad. Japan*, **12**, 221, 225, 277.
Hasegawa, M. 1940 *Trans. Washington Meeting A.T.M.E., Bull.*, no. 11, p. 311, Edinburgh.
Hasegawa, M. 1960 *J. Geophys. Res.* **65**, 1437.
Hasegawa, M. & Ota, M. 1950 *Trans. Oslo Meeting A.T.M.E., Bull.*, no. 13, pp. 426, 431. Washington.
Howe, H. H. 1950 *J. Geophys. Res.* **55**, 271, 1950.
Jeffreys, H. & Jeffreys, B. S. 1950 *Methods of mathematical physics.* Cambridge University Press.
Johnston, H. F. & McNish, A. G. 1932 *C.R. Congr. Int. Electricité*, **12**, 41. Paris.
Kertz, W. 1954 *Nachr. Akad. Wiss. Göttingen. Math.-phys. Kl.* (*Abt. Math. Phys. Chem.*), p. 101.
McNish, A. G. 1937 *Trans. Edinburgh Meeting A.T.M.E., Bull.*, no. 10, 271. Copenhagen.
Maeda, H. 1953 *J. Geomagn. Geoelect., Kyoto*, **5**, 39.
Matsushita, S. 1960 *J. Geophys. Res.* **65**, 3835.
Nagata, T. & Mizuno, H. 1955 *J. Geomagn. Geoelect., Kyoto*, **7**, 69.
Onwumechilli, C. A. 1959 *J. Atmos. Terr. Phys.* **13**, 222, 235.
Price, A. T. & Wilkins, G. A. 1951 *J. Geophys. Res.* **56**, 259.
Romana, A. & Cardus, J. O. 1955 *Obs. del. Ebro. Misc.*, no. Publ. 14, Tarragona.
Rooney, W. J. 1945 *Terr. Magn. Atmos. Elect.* **50**, 175.
Siebert, M. & Kertz, W. 1957 *Nachr. Akad. Wiss. Göttingen, Math.-phys. Kl.* (*Abt. Math. Phys. Chem.*), p. 87.
Taylor, J. H. 1944 *Terr. Magn. Atmos. Elect.* **49**, 223.
Vestine, E. H. 1941 *Terr. Magn. Atmos. Elect.* **46**, 27.
Wilkins, G. A. 1951 Ph.D. Thesis, University of London.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES